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Mistreatment of women and children: Analysis by mathematics of uncertainty

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"This paper is dedicated to Professor Mohammad Mehdi Zahedi on the occasion of his 70th birthday"

Abstract

We examine the mistreatment of women and children using fuzzy implication operators. We show how fuzzy implication operators can be used to define fuzzy similarity measures. These fuzzy similarity measures are used to compare the similarity of various rankings of countries with respect to the security status, gender equality and human development of women and children. We find a medium to high similarity between these categories.

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1 Introduction

We examine the mistreatment of women and children using fuzzy implication operators. We show how fuzzy implication operators can be used to define fuzzy similarity measures. These fuzzy similarity measures are used to compare the similarity of various rankings of countries with respect to the security status, gender equality and human development of women and children. We find a medium to high similarity between these categories. We prove relationships between certain fuzzy implication operators.

The following is taken from [12]. Technology-facilitated violence: Available evidence collected at country and regional levels confirms high prevalence rates against women and girls. One in 10 women in the European union has experienced cyber-harassment since the age of 15. In the Arab States, a regional study found that 60 percent of women internet users in the region had been exposed to online violence. In Uganda , in 2021, 49% of women reported being involved in online harassment at some points in their lifetime. According to a 2016 survey by the Korean National Human Rights Commission, 85 percent of women experienced hate speech online.

Climate change and violence: Climate change and slow environmental degradation exacerbate the risks of violence against women and girls due to displacement, resource scarcity and food insecurity and disruption to service provision for survivors. Following Hurricane Katrina in 2005, the rate of rape among women displaced to trailer parks rose 53.6 times the baseline rate in Mississippi, USA, for that rate. In Ethiopia there was an increase in girls sold into early marriage in exchange for livestock to help families cope with the impacts of prolonged droughts. Nepal witnessed an increase in trafficking from an estimated 3,00-5,000 annually in 1990 to 12,000-20,000 per year after the 2015 earthquake.

Trafficking in women: In 2020, for every 10 victims of human trafficking detected globally, about four were women and about two were girls. Most of the detected victims of trafficking for sexual exploitation (91 percent) are women.

The study in [12] also considered femicides/feminicides, prevalence of violence against women and girls, impact of COVID-19 on violence against women and girls, reporting violence against women, laws on violence against women and girls, and many other categories.

The following is from [13]. Gender-based violence occurs in every country in the world and across all economic and social groups. One in three women and girls will experience sexual or physical violence in their lifetimes. Gender-based violence has been ingrained into society, in some countries and regions more than others. In many communities, violence against girls and women is expected and even accepted. The military use of schools continues in Syria, Yemen, Sudan, the Philippines and Afghanistan. In some contexts, schoolgirls have been specifically targeted for sexual violence and by armed groups who oppose female education. Some global trends are 15 million girls are married before the age of 18, 30 million girls are at risk of female genital mutilation in the next decade, 1 in 3 girls and women live in countries where marital rape is not an explicit crime. Due to their gender, girls are often forced to drop out of school, are prevented from accessing income-generating opportunities, and ultimately face social exclusion. More information can be found in [13]. See also [2, 8, 17].

The Women, Peace and Security (WPS) Index ranks 177 countries and economies on women's status, [16]. Countries are also ranked according to their achievement of the rights of a child, [15]. The Gender Inequality Index (GII) ranks countries with respect to the loss of achievement within a country due to gender inequality, [14]. The Human Development Index (GHI) ranks countries with respect to human development. In this paper, we determine the similarity of the rankings using various fuzzy similarity measures.

Let X be a set. Then the fuzzy power set of X, denoted $\mathcal{FP}(X)$, is the set of all fuzzy subsets of X. Define the relations $\vee, \wedge$ on the closed interval $[0,1]$ by for all $a, b \in [0,1], a \vee b$ is the maximum of a and b and $a \wedge b$ is the minimum of a and b.

Define $\overline{\wedge} : [0,1] \times [0,1] \to [0,1]$ by $\overline{\wedge}(a,b) = 1$ if $a = b$ and $a \wedge b$ if $a \neq b$. Define $\varnothing : [0,1] \times [0,1] \to [0,1]$ by $\varnothing(a,b) = 1$ if $a = b$ and $\frac{a \wedge b}{a \vee b}$ if $a \neq b$. Note that for all $a, b \in [0,1]$, $\varnothing(a,b) = \frac{a \wedge b}{a \vee b}$.

2 Preliminary results

Definition 2.1. *Let S be a function of $\mathcal{FP}(X) \times \mathcal{FP}(X)$ into $[0,1]$. Then S is called a fuzzy similarity measure on $\mathcal{FP}(X)$ if the following properties hold: $\forall \mu, \nu, \rho \in \mathcal{FP}(X)$:*

(1) $S(\mu, \nu) = S(\nu, \mu)$;

(2) $S(\mu, \nu) = 1$ *if and only if* $\mu = \nu$;

(3) *If* $\mu \subseteq \nu \subseteq \rho$, *then* $S(\mu, \rho) \leq S(\mu, \nu) \wedge S(\nu, \rho)$;

(4) *If* $S(\mu, \nu) = 0$, *then* $\forall x \in X, \mu(x) \wedge \nu(x) = 0$.

Example 2.2. *Let* μ, ν *be fuzzy subsets of a set* X. *Then* M *and* S *are fuzzy similarity measures on* $\mathcal{FP}(X)$, *where*

$$\begin{aligned} M(\mu, \nu) &= \frac{\sum_{x \in X} \mu(x) \wedge \nu(x)}{\sum_{x \in X} \mu(x) \vee \nu(x)}, \\ S(\mu, \nu) &= 1 - \frac{\sum_{x \in X} |\mu(x) - \nu(x)|}{\sum_{x \in X} (\mu(x) + \nu(x))}. \end{aligned}$$

Results concerning fuzzy similarity measures can be found in [10, 6].

Definition 2.3 (1, p. 14)**.** *Let* I *be a function of* $[0, 1] \times [0, 1]$ *into* $[0, 1]$ *such that* $I(0, 0) = I(0, 1) = I(1, 1) = 1$ *and* $I(1, 0) = 0$. *Then* I *is called an implication operator.*

An implication operator I is said to satisfy the identity principle if $I(x, x) = 1$ for all $x \in [0, 1]$. An implication operator is said to satisfy the ordering principle if $x \leq y \Leftrightarrow I(x, y) = 1$, [3]. Clearly, the ordering principle implies the identity principle.

An implication operator I is called a hybrid monotonous implication operator if $I(x, \_)$ is non decreasing for all $x \in [0, 1]$ and $I(\_, y)$ is nonincreasing for all $y \in [0, 1]$, [11].

I_1, I_2, and L defined in the following example are hybrid monotonous implication operators that satisfy the ordering principle.

Example 2.4. *Let* $x, y \in [0, 1]$.

(1) *Gödel implication operator:* $I_1(x, y) = 1$ *if* $x \leq y$, $I_1(x, y) = y$ *otherwise,* [4].

(2) *Goguen implication operator:* $I_2(x, y) = 1$ *if* $x \leq y$ *and* $I_2(x, y) = y/x$ *otherwise,* [9].

(3) *Łukasiewicz implication operator:* $L(x, y) = (1 - x + y) \wedge 1$, [5].

Let X be a set with n elements, $n > 1$, say $X = \{x_1, \cdots, x_n\}$. Let A be one-to-one function of X onto $\{1, \cdots, n\}$. Then A is called a ranking of X. Define the fuzzy subset μ_A of X by for all $x \in X, \mu_A(x) = \frac{A(x)}{n}$. Then μ_A is called the fuzzy subset associated with A.

Let A be the ranking $1, 2, \cdots, n$ and B be the ranking $n, \cdots, 2, 1$. For n even, we have

$$I_1(\mu_A, \mu_B) = \frac{1}{n}(1 + \cdots + 1 + \frac{n}{2} + \cdots + 2 + 1)\frac{1}{n},$$

and

$$I_2(\mu_A, \mu_B) = \frac{1}{n}(1 + \cdots + 1 + \frac{\frac{n}{2}}{\frac{n+2}{2}} + \cdots + \frac{2}{n-1} + \frac{1}{n}).$$

For n odd, we have

$$I_1(\mu_A, \mu_B) = \frac{1}{n}(1 + \cdots + 1 + \frac{n+1}{2} + \cdots + 2 + 1)\frac{1}{n},$$

and

$$I_2(\mu_A, \mu_B) = \frac{1}{n}(1 + \cdots + 1 + \frac{\frac{n-1}{2}}{\frac{n+3}{2}} + \cdots + \frac{2}{n-1} + \frac{1}{n}).$$

By [7, Theorem 3.1], S_L is a fuzzy similarity, where

$$S_L(\mu_A, \mu_B) = \frac{1}{n} \sum_{x \in X} (1 - \mu_A(x)) \wedge (1 - \mu_B(x)) \wedge \mu_A(x) \wedge \mu_B(x).$$

Definition 2.5. [1] *Let I be an implication operator. Define the fuzzy subset E_I of $\mathcal{FP}(X) \times \mathcal{FP}(X)$ by for all $\mu, \nu \in \mathcal{FP}(X)$,*

$$E_I(\mu, \nu) = \wedge\{\wedge\{I(\mu(x), \nu(x)) | x \in X\}, \wedge\{I(\nu(x), \mu(x)) | x \in X\}\}.$$

Then $E_I(\mu, \nu)$ is called the degree of sameness of μ and ν.

Proposition 2.6. *Let I be a hybrid monotonous implication operator that satisfies the ordering principle. Then E_I satisfies the following properties $\forall \mu, \nu, \rho \in \mathcal{FP}(X)$:*

(1) *$E_I(\mu, \nu) = E_I(\nu, \mu)$;*

(2) *$E_I(\mu, \nu) = 1$ implies $\mu = \nu$;*

(3) *If $\mu \subseteq \nu \subseteq \rho$, then $E_I(\mu, \rho) \leq E_I(\mu, \nu) \wedge E_I(\nu, \rho)$.*

Proof. (1) Clearly, $E_I(\mu, \nu) = E_I(\nu, \mu)$.

(2) $E_I(\mu, \nu) = 1 \Leftrightarrow \wedge\{I(\mu(x), \nu(x)) | x \in X\} = 1$ and $\wedge\{I(\nu(x), \mu(x)) | x \in X\}\} = 1 \Leftrightarrow \mu(x) = \nu(x) = 1\ \forall x \in X \Rightarrow \mu = \nu$.

(3) Suppose $\mu \subseteq \nu \subseteq \rho$. Then, for any $x \in X$, $\mu(x) \leq \nu(x) \leq \rho(x)$. Thus for any $x \in X$, $I(\mu(x), \nu(x)) = 1, I(\mu(x), \rho(x)) = 1$, and $I(\nu(x), \rho(x))) = 1$ since I satisfies the ordering principle. Hence $I(\mu, \nu) = 1$. Now, for any $x \in X$, $I(\rho(x), \mu(x)) \leq I(\nu(x), \mu(x))$ and $I(\rho(x), \mu(x)) \leq I(\rho(x), \nu(x))$ since I is hybrid monotonous. Now

$$\begin{aligned} E_I(\mu, \rho) &= 1 \wedge (\wedge\{I(\rho(x), \mu(x)) | x \in X\}) \leq 1 \wedge (\wedge\{I(\nu(x), \mu(x)) | x \in X\}) &= E_I(\mu, \nu), \\ E_I(\mu, \rho) &= 1 \wedge (\wedge\{I(\rho(x), \mu(x)) | x \in X\}) \leq 1 \wedge (\wedge\{I(\rho(x), \nu(x) | x \in X\}) &= E_I(\nu, \rho). \end{aligned}$$

□

In [7], the following definition was used for defining fuzzy similarity measures from implication operators.

Definition 2.7. *Let I be an implication operator. Define $S_I : \mathcal{FP}(X) \times \mathcal{FP}(X) \to [0, 1]$ by for all $(\mu, \nu) \in \mathcal{FP}(X) \times \mathcal{FP}(X)$,*

$$S_I(\mu, \nu) = \frac{1}{n} \sum_{x \in X} I((\mu(x), \nu(x)) \wedge I((\nu(x), \mu(x))).$$

Then S_I is called a degree of likeness.

In [7, Theorem 2.7], it was shown that the function S of Definition 2.7 is a fuzzy similarity measure. Other implication operators can be found in [1].

We determine the smallest value a fuzzy similarity measure can be with respect to rankings since then the ration $(S - \min)/(\max - \min)$ provides a similarity measure that ranges from 0 to 1.

For $E_I(\mu_A, \mu_B)$, (4) of Definition 2.1 holds vacuously for rankings A and B since $E_I(\mu_A, \mu_B)$ is never 0. (There does not exist $x \in X$ such that $\mu_A(x) = 0$ or $\mu_B(x) = 0$.)

For two rankings A and B of X, $\sum_{x\in X}(A(x)+B(x)) = n(n+1)$ and so $\sum_{x\in X}(\mu_A(x)+\mu_B(x)) = n+1$. Thus for S of Example 2.2,

$$S(\mu_A, \mu_B) = 1 - \frac{\sum_{x\in X} |\mu_A(x) - \nu_B(x)|}{n+1}.$$

3 Gödel and Goguen implication operators

Recall that I_1 and I_2 below are defined in Example 2.4.

Theorem 3.1. (1) *Suppose n is even. Let A be the ranking: $1, 2, \cdots \frac{n}{2}, \frac{n+2}{2} \cdots, n-1, n$ and let B be the ranking $n, n-1, \cdots, \frac{n+2}{2}, \frac{n}{2}, \cdots, 2, 1$. Then,*

$$I_2(\mu_A, \mu_B) = I_1(\mu_A, \mu_B) + \frac{1}{n}((n+1)(\sum_{j=\frac{n}{2}+1}^{n} \frac{1}{j}) - \frac{n}{2}) - (\frac{1}{8} + \frac{1}{2n^2}).$$

(2) *Suppose n is odd. Let A be the ranking $1, 2, \cdots, \frac{n+1}{2}, \cdots, n-1, n$ and B be the ranking $n, n-1, \cdots, \frac{n+1}{2}, \cdots, 2, 1$. Then,*

$$I_2(\mu_A, \mu_B) = I_1(\mu_A, \mu_B) + \frac{1}{n}((n+1)(\sum_{j=\frac{n+3}{2}}^{n} \frac{1}{j}) - \frac{n-1}{2}) - (\frac{1}{8} - \frac{1}{8n^2}).$$

Proof. (1) We have that

$$\begin{aligned}
I_1(\mu_A, \mu_B) &= \frac{1}{n}(1 + \cdots + 1 + \frac{1}{n}(\frac{n}{2} + \cdots + 2 + 1)),\\
I_2(\mu_A, \mu_B) &= \frac{1}{n}(1 + \cdots + 1 + (\frac{\frac{n}{2}}{\frac{n+2}{2}} + \cdots + \frac{2}{n-1} + \frac{1}{n}))\\
&= \frac{1}{n}(1 + \cdots + 1 + (\frac{\frac{n}{2}}{\frac{n}{2}+1} + \cdots + \frac{2}{n-1} + \frac{1}{n})).
\end{aligned}$$

Hence,

$$I_2(\mu_A, \mu_B) = I_1(\mu_A, \mu_B) + \frac{1}{n}(\frac{\frac{n}{2}}{\frac{n}{2}+1} + \cdots + \frac{2}{n-1} + \frac{1}{n})) - \frac{1}{n^2}(\frac{n}{2} + \cdots + 2 + 1).$$

Thus,

$$\begin{aligned}
I_2(\mu_A, \mu_B) &= I_1(\mu_A, \mu_B) + \frac{1}{n}(\frac{\frac{n}{2}}{\frac{n}{2}+1} + \cdots + \frac{2}{n-1} + \frac{1}{n})) - \frac{1}{n^2}((\frac{n}{2})(\frac{n}{2}+1)\frac{1}{2})\\
&= I_1(\mu_A, \mu_B) + \frac{1}{n}\sum_{i=1}^{\frac{n}{2}} \frac{i}{n-i+1} - (\frac{1}{8} + \frac{1}{2n^2}).
\end{aligned}$$

Let $j = n - i + 1$. Then $i = n - j + 1$ and $j = n, n-1, \cdots, \frac{n}{2} + 1$. Now

$$\begin{aligned}\sum_{i=1}^{\frac{n}{2}} \frac{i}{n-i+1} &= \sum_{j=\frac{n}{2}+1}^{n} \frac{n-j+1}{j} = \sum_{j=\frac{n}{2}+1}^{n} (\frac{n}{j} - 1 + \frac{1}{j}) \\ &= (n+1)(\sum_{j=\frac{n}{2}+1}^{n} \frac{1}{j}) - \frac{n}{2}. \qquad (3.1)\end{aligned}$$

(2) We have that,

$$\begin{aligned}I_1(\mu_A, \mu_B) &= \frac{1}{n}(1 + \cdots + 1 + \frac{1}{n}(\frac{n-1}{2} + \cdots + 2 + 1)), \\ I_2(\mu_A, \mu_B) &= \frac{1}{n}(1 + \cdots + 1 + (\frac{\frac{n-1}{2}}{\frac{n+3}{2}} + \cdots + \frac{2}{n-1} + \frac{1}{n})).\end{aligned}$$

Thus,

$$\begin{aligned}I_2(\mu_A, \mu_B) &= I_1(\mu_A, \mu_B) + \frac{1}{n}(\frac{\frac{n-1}{2}}{\frac{n+3}{2}} + \cdots + \frac{2}{n-1} + \frac{1}{n}) - \frac{1}{n^2}(\frac{n-1}{2} + \cdots + 2 + 1) \\ &= I_1(\mu_A, \mu_B) + \frac{1}{n}\sum_{i=1}^{\frac{n-1}{2}} \frac{i}{n-i+1} - \frac{1}{n^2}(\frac{n-1}{2})(\frac{n-1}{2} + 1)\frac{1}{2}) \\ &= I_1(\mu_A, \mu_B) + \frac{1}{n}\sum_{i=1}^{\frac{n-1}{2}} \frac{i}{n-i+1} - (\frac{1}{8} - \frac{1}{8n^2}).\end{aligned}$$

Let $j = n - i + 1$. Then $i = n - j + 1$ and $j = n, n-1, \cdots, \frac{n}{2} + \frac{3}{2}$. Now

$$\begin{aligned}\sum_{i=1}^{\frac{n-1}{2}} \frac{i}{n-i+1} &= \sum_{j=\frac{n+3}{2}}^{n} \frac{n-j+1}{j} = \sum_{j=\frac{n+3}{2}}^{n} (\frac{n}{j} - 1 + \frac{1}{j}) \\ &= (n+1)(\sum_{j=\frac{n+3}{2}}^{n} \frac{1}{j}) - \frac{n-1}{2}. \qquad (3.2)\end{aligned}$$

□

We next determine approximate values for $\sum_{j=\frac{n}{2}+1}^{n} \frac{1}{j}$ when n is even and $\sum_{j=\frac{n+3}{2}}^{n} \frac{1}{j}$ when n is odd. These approximate values appear in the proof of the following theorem. We recall that $H_n = \sum_{j=1}^{n} \frac{1}{j}$ is a harmonic sum which sums approximately to $\gamma + \ln n$, where γ is the Euler-Mascheroni constant, $\gamma \approx 0.5772$, where $\approx$ denotes approximately equal to.

Theorem 3.2. (1) *Suppose n is even. Let A be the ranking: $1, 2, \cdots \frac{n}{2}, \frac{n+2}{2} \cdots, n-1, n$ and let B be the ranking $n, n-1, \ldots, \frac{n+2}{2}, \frac{n}{2}, \ldots, 2, 1$. Then*

$$I_2(\mu_A, \mu_B) \approx I_1(\mu_A, \mu_B) + \ln 2 - \frac{5}{8} + \frac{1}{n}\ln 2 - \frac{1}{2n^2}.$$

(2) *Suppose n is odd. Let A be the ranking $1, 2, \ldots, \frac{n+1}{2}, \ldots, n-1, n$ and B be the ranking $n, n-1, \ldots, \frac{n+1}{2}, \ldots, 2, 1$. Then*

$$I_2(\mu_A, \mu_B) \approx I_1(\mu_A, \mu_B) + \ln 2 - \frac{5}{8} + \ln\frac{n}{n+1} + \frac{1}{n}\ln\frac{2n}{n+1} + \frac{1}{2n} + \frac{1}{8n^2}.$$

Proof. (1) Let n be even. Consider $\sum_{j=\frac{n}{2}+1}^{n}\frac{1}{j}$. We have $\sum_{j=\frac{n}{2}+1}^{n}\frac{1}{j} = \sum_{j=1}^{n}\frac{1}{j} - \sum_{j=1}^{\frac{n}{2}}\frac{1}{j} \approx \gamma + \ln n - (\gamma + \ln\frac{n}{2}) = \ln n - \ln\frac{n}{2} = \ln 2$. Thus, by Theorem 3.1 (2) and equation (3.1),

$$\begin{aligned} I_2(\mu_A, \mu_B) &\approx I_1(\mu_A, \mu_B) + \frac{1}{n}((n+1)\ln 2 - \frac{n}{2}) - (\frac{1}{8} + \frac{1}{2n^2}) \\ &= I_1(\mu_A, \mu_B) + \ln 2 - \frac{5}{8} + \frac{1}{n}\ln 2 - \frac{1}{2n^2}. \end{aligned}$$

(2) Let n be odd. Consider $\sum_{j=\frac{n+3}{2}}^{n}\frac{1}{j}$. We have $\sum_{j=\frac{n+3}{2}}^{n}\frac{1}{j} = \sum_{j=1}^{n}\frac{1}{j} - \sum_{j=1}^{\frac{n+1}{2}}\frac{1}{j} \approx \gamma + \ln n - (\gamma + \ln\frac{n+1}{2}) = \ln n - \ln\frac{n+1}{2} = \ln\frac{2n}{n+1}$. Thus, by Theorem 3.1 (2) and equation (3.2),

$$\begin{aligned} I_2(\mu_A, \mu_{B^-}) &= I_1(\mu_A, \mu_B) + \frac{1}{n}((n+1)(\sum_{j=\frac{n+3}{2}}^{n}\frac{1}{j}) - \frac{n-1}{2}) - (\frac{1}{8} - \frac{1}{8n^2}) \\ &\approx I_1(\mu_A, \mu_B) + \frac{1}{n}((n+1)\ln\frac{2n}{n+1} - \frac{n-1}{2}) - (\frac{1}{8} - \frac{1}{8n^2}) \\ &= I_1(\mu_A, \mu_B) + (1 + \frac{1}{n})\ln\frac{2n}{n+1} - \frac{1}{2} + \frac{1}{2n} - \frac{1}{8} + \frac{1}{8n^2} \\ &= I_1(\mu_A, \mu_B) + \ln\frac{2n}{n+1} + \frac{1}{n}\ln\frac{2n}{n+1} - \frac{5}{8} + \frac{1}{2n} + \frac{1}{8n^2} \\ &= I_1(\mu_A, \mu_B) + \ln 2 - \frac{5}{8} + \ln\frac{n}{n+1} + \frac{1}{n}\ln\frac{2n}{n+1} + \frac{1}{2n} + \frac{1}{8n^2}. \end{aligned}$$

□

Example 3.3. *Consider S_1. Let $n = 3$. Let A be the ranking $1, 2, 3$ and B be the ranking $3, 2, 1$. Then*

$$\begin{aligned} S_1(\mu_A, \mu_B) &= \frac{1}{3}(I_1(\frac{1}{3}, \frac{3}{3}) \wedge I_1(\frac{3}{3}, \frac{1}{3}) + I_1(\frac{2}{3}, \frac{2}{3}) \wedge I_1(\frac{2}{3}, \frac{2}{3}) + I_1(\frac{3}{3}, \frac{1}{3}) \wedge I_1(\frac{1}{3}, \frac{3}{3})) \\ &= \frac{1}{3}(\frac{1}{3} + 1 + \frac{1}{3}) = \frac{5}{9}. \end{aligned}$$

Let C be the ranking $3, 1, 2$. Then

$$\begin{aligned} S_1(\mu_A, \mu_C) &= \frac{1}{3}(I_1(\frac{1}{3}, \frac{3}{3}) \wedge I_1(\frac{3}{3}, \frac{1}{3}) + I_1(\frac{2}{3}, \frac{1}{3}) \wedge I_1(\frac{1}{3}, \frac{2}{3}) + I_1(\frac{3}{3}, \frac{2}{3}) \wedge I_1(\frac{2}{3}, \frac{3}{3})) \\ &= \frac{1}{3}(\frac{1}{3} + \frac{1}{3} + \frac{2}{3}) = \frac{4}{9}. \end{aligned}$$

Hence for n odd, the rankings μ_A and μ_B do not give the smallest value S_1 can be.

Example 3.4. *Consider I_1. Let $n = 6$, A be the ranking $1, 2, 3, 4, 5, 6$ and B be the ranking $6, 5, 4, 3, 2, 1$. Then $I_1(\mu_A, \mu_B) = \frac{1}{6}(1 + 1 + 1 + \frac{3}{6} + \frac{2}{6} + \frac{1}{6}) = \frac{24}{36}$. Let C be the ranking $6, 5, 2, 3, 1, 4$. Then $I_1(\mu_A, \mu_C) = \frac{1}{6}(1 + 1 + \frac{2}{6} + \frac{3}{6} + \frac{1}{6} + \frac{4}{6}) = \frac{22}{36}$. Hence even though the rankings A and B yield the smallest S_1, it is not the case that A and B yield the smallest I_1.*

Theorem 3.5. *Let n be odd. Let A be the ranking $1, 2, \cdots, \frac{n+1}{2}, \cdots, n-1, n$ and B the ranking $n, n-1, \cdots, \frac{n+1}{2}, \cdots, 2, 1$. Let C be the ranking obtain from B by interchanging the middle and the middle plus next term. Then $S_1(\mu_A, \mu_C) = \frac{1}{4} + \frac{1}{2n} - \frac{1}{4n^2}$ is the smallest value S_1 can be.*

Proof.

$$\begin{aligned} S_1(\mu_A, \mu_C) &= \frac{1}{n}(n - \frac{n-1}{2} + 2(1 + 2 + \cdots + \frac{n-1}{2}))\frac{1}{n} \\ &= (1 - \frac{1}{2} + \frac{2}{n}((\frac{n-1}{2})(\frac{n+1}{2}))\frac{1}{2})\frac{1}{n} \\ &= \frac{1}{n} - \frac{1}{2n} + \frac{1}{2n^2} + \frac{n^2-1}{4n^2} \\ &= \frac{1}{2n} + \frac{1}{2n^2} + \frac{1}{4} - \frac{1}{4n^2} \\ &= \frac{1}{4} + \frac{1}{2n} - \frac{1}{4n^2}. \end{aligned}$$

This is the smallest value S_1 can be since the term $2(1+2+\cdots+\frac{n-1}{2})$ represents the smallest element in the ranking while the $n - \frac{n-1}{2}$ term represents the next smallest. □

For example, let $n = 7$. Then $A : 1, 2, 3, 4, 5, 6, 7$ and $B : 7, 6, 5, 4, 3, 2, 1$ and $C : 7, 6.5, 3, 4, 2, 1$ Now

$$\begin{aligned} S_1(\mu_A, \mu_B) &= \frac{1}{7}(\frac{1}{7} + \frac{2}{7} + \frac{3}{7} + 1 + \frac{3}{7} + \frac{2}{7} + \frac{1}{7}), \\ S_1(\mu_A, \mu_C) &= \frac{1}{7}(\frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{4}{7} + \frac{3}{7} + \frac{2}{7} + \frac{1}{7}). \end{aligned}$$

The middle term becomes $\frac{n - \frac{n-1}{2}}{n}$ instead of $\frac{n}{n}$.

Theorem 3.6. *Let A be the ranking $1, 2, \cdots, n$ and C be the ranking $n, 1, 2, \cdots, n-1$. Then $I_1(\mu_A, \mu_C) = \frac{1}{n}(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1})$ is the smallest value I_1 can be.*

Proof. In any two μ, ν, only 1 and the values of ν can appear in $I_1(\mu, \nu)$. Now for rankings, $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n-1}$ are the smallest. None of these can appear more than once although 1 may appear more than once. □

Example 3.7. *Let A be the ranking $1, 2, 3, 4, 5, 6$ and let C be the ranking $6, 1, 2, 3, 4, 5$. Then $I_1(\mu_A, \mu_C) = \frac{1}{6}(1 + \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6}) = \frac{21}{36}$. By Theorem 3.6, this is the smallest I_1 can be. Now $S_1(\mu_A, \mu_C) = \frac{1}{6}(\frac{1}{6} + \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6}) = \frac{16}{6}$. Let A be the ranking $1, 2, 3, 4, 5, 6$ and B the ranking $6, 5, 4, 3, 2, 1$. Then $S_1(\mu_A, \mu_B) = \frac{12}{36}$ is the smallest S_1 can be. Hence the smallest I_1 can be doesn't yield the smallest S_1 can be.*

4 Women and children and similarity results

The Women, Peace and Security (WPS) Index ranks 177 countries and economies on women's status. As the only index to bring together indicators of women's inclusion, justice and security, the WPS Index is a valuable measure of women's status that can be used to track trends, guide policy making, and hold governments accountable for their promises to advance women's rights and opportunities, [16].

The WPS Index reveals glaring disparities around the world. All countries on the index have room for improvement, and many perform considerably better or worse on some indicators of women's status than in others [16].

The application, implementation, and interpretation of the 8 Fundamental Rights of a child are guided and determined by 4 Guiding Principles of the Convention on the Rights of the Child; the principle of nondiscrimination, the "best interests of the child", the principle of life, the survival, and development, and the principle of inclusion and participation, [15].

Right to Life: The right to life means that each child must be able to live his or her life. Children have the right not to be killed. They have the right to survive and grow up in proper conditions.

Right to Education: The right to education allows each child to receive, to enjoy a social life, and to build his or her own future. The right is essential for economic, social and cultural development.

Right to Food: The right to food is the right of each child to eat. It is the right to not die of hunger and not to suffer from malnutrition. Every five seconds, a child dies of hunger somewhere in the world.

Right to Health: The right to health means that children must be protected against illness. They must be allowed to grow and become healthy adults. This contributes to developing an active society.

Right to Water: The right to water means children have the right to safe drinking water and proper sanitary conditions. The right to water is essential for good health, survival and proper growth.

Right to Identity: Each child has the right to have a surname, a first name, a nationality, and to know who his or her relatives are. The right to identity also means that each child's existence and rights must be officially recognized.

Right to Freedom: The right to liberty is the child's right to express him or herself, to have opinions, to have access to information, and to participate in decisions which affect his or her life. Children also have the right to religious freedom.

Right to Protection: The right to protection is the right to live in a secure and protective environment which preserves the child's well-being. Each child has the right to be protected from all forms of mistreatment, discrimination, and exploitation.

The Gender Inequality Index (GII) is an index for the measurement of gender disparity that was introduced in the 2010 Human development report. According to United Nations Development Programme (UNDP), this index is a composite measure to quantify loss of achievement within a country due to gender inequality. It uses three dimensions to measure opportunity cost: reproductive health, empowerment, and labor market participation, [14]. The Human Development Index (HDI) provides a composite measure of human development used by the UNDP, [14].

We next provide the scores for the region, Arab States.

Table 1: Country Scores

Country	GII	Peace& Security	HDI	Rights
Angola	132	127	148	177
Benin	148	138	158	165
Botswana	116	104	100	136

Table 1: Country Scores(cont.)

Country	GII	Peace& Security	HDI	Rights
Burkina Faso	147	158	182	185
Burundi	124	172	185	174
Cabo Verde	89	64	126	74
Cameroon	141	161	153	172
C. African Rep.	159	175	188	190
Chad	160	163	187	196
Comoros		148		160
Congo	144	150	149	166
Congo, Dem. Rep.	150	174	175	193
Cote d'lvoire		136		171
Djibouti		153		153
Equatorial Guinea		119		179
Eritrea				169
Eswatini	143	170	138	
Ethiopia	125	146	173	187
Gabon	128	131	119	142
Gambia	148	135	172	173
Ghana	135	108	138	147
Guinea		145		186
Guinea-Bissau		156		188
Kenya	126	149	143	157
Lesotho	139	125	165	159
Liberia	156	154	175	180
Madagascar		152		151
Malawi	142	146	174	155
Mali	158	158	184	191
Mauritania	151	151	157	176
Mauritius	78	93	66	41
Mozambique	127	134	181	178
Namibia	106	122	130	138
Niger	154	166	189	192
Nigeria		162		182
Rwanda	92	103	160	164
S. Torre and Prin.	133	109	135	144
Senegal	130	119	168	168
Seycheles		43		52
Sierra Leone	155	144	182	189
Somolia		169		195
S. Africa	93	91	114	131
S. Sudan		173		184
Sudan	138	164	170	183
Tanzania	140	107	163	167
Togo	145	128	167	150
Uganda	131	143	159	175

Table 1: Country Scores(cont.)

Country	GII	Peace& Security	HDI	Rights
Zambia	137	141	146	181
Zimbabwe	129	126	150	161

We rank the entries in the above table. Let μ_A be the Peace and Security rank and μ_B be the Rights rank. We find that

$$\begin{aligned} S(\mu_A, \mu_B) &= 1 - \frac{372}{(47)(48)} = 0.835, \\ M(\mu_A, \mu_B) &= \frac{0.835}{2 - 0.835} = 0.717, \\ S_L(\mu_A, \mu_B 0 &= 0.835 + \frac{1}{47}(-0.165) = 0.831, \\ S_1(\mu_A, \mu_B) &= \frac{1}{47}(\frac{943}{47}) = 0.427, \\ S_2(\mu_A, \mu_B) &= \frac{1}{47}(32.341) = 0.688. \end{aligned}$$

The smallest S can be is $\frac{n/2+1}{n+1}$ if n is even and $\frac{1}{2} + \frac{1}{2n}$ if n is odd.
The smallest M can be is $\frac{n+2}{3n+2}$ if n is even and $\frac{n+1}{3n-1}$ if n is odd.
The smallest S_L can be is $\frac{3}{4} - \frac{1}{n} - \frac{1}{n^2}$ if n is even and $\frac{1}{2} + \frac{1}{2n^2}$ if n is odd.
The smallest S_1 can be is $\frac{1}{4} + \frac{1}{2n}$ if n is even and $\frac{1}{4} + \frac{3}{4n^2}$ if n is odd.
The smallest S_2 can be is approximately $0.386 + \frac{2}{n} \ln 2$ if n is even and approximately $2 \ln \frac{2n}{n+1} + \frac{2}{n} \ln \frac{2n}{n+1} - 1 + \frac{2}{n}$ if n is odd.

Here $n = 47$ is odd. Thus we obtain the following smallest values:

$$\begin{aligned} S(\mu_A, \mu_B) &= 0.500 + 011 = 0.511, \\ M(\mu_A, \mu_B) &= \frac{48}{140} = 0.343, \\ S_L(\mu_A, \mu_B 0 &= 0.500, \\ S_1(\mu_A, \mu_B) &= 0.250, \\ S_2(\mu_A, \mu_B) &\approx 0.418. \end{aligned}$$

Thus

$$\begin{aligned} \frac{0.835 - 0.511}{1 - 0.511} &= \frac{0.324}{0.489} = 0.663, \\ \frac{0.717 - 0.343}{1 - 0.343} &= \frac{0.374}{0.657} = 0.569, \\ \frac{0.831 - 0.500}{1 - 0.500} &= \frac{0.331}{0.500} = 0.663, \\ \frac{0.427 - 0.250}{1 - 0.250} &= \frac{0.177}{0.750} = 0.236, \\ \frac{0.688 - 0.418}{1 - 0.418} &= \frac{0.270}{0.582} = 0.564. \end{aligned}$$

Disregarding S_1, we see that the fuzzy similarity measures range from medium to high for the Peace and Security rank and the Rights rank.

We next consider GII, HDI, and Peace and Security. We delete the countries in Table 1 whose entries in columns 2 and 4 for which there were no entries in columns 1 and 3. We then reranked the scores of the remaining countries.

Let μ_C be the GII rank and μ_D be the HDI rank. We find that

$$\begin{aligned}
S(\mu_C,\mu_D) &= 1-\frac{245}{(37)(88)}=0.826,\\
M(\mu_C,\mu_D) &= \frac{0.826}{2-0.826}=0.704,\\
S_L(\mu_C,\mu_D) &= 0.826+\frac{1}{37}(-0.174)=0.821,\\
S_1(\mu_C,\mu_D) &= \frac{1}{37}(\frac{580.5}{37})=0.424,\\
S_2(\mu_C,\mu_D) &= \frac{1}{37}(25.23)=0.682.
\end{aligned}$$

Here $n=37$ is odd. Thus from the above statements concerning smallest values, we obtain the following smallest values:

$$\begin{aligned}
S(\mu_C,\mu_D) &= 0.514,\\
M(\mu_C,\mu_D) &= \frac{38}{110}=0.345,\\
S_L(\mu_C,\mu_D) &= 0.500,\\
S_1(\mu_C,\mu_D) &= 0.251,\\
S_2(\mu_C,\mu_D) &\approx 0.436.
\end{aligned}$$

$$\begin{aligned}
\frac{0.826-0.514}{1-0.514} &= \frac{0.312}{0.486}=0.642,\\
\frac{0.704-0.345}{1-0.345} &= \frac{0.359}{0.655}=0.548,\\
\frac{0.821-0.500}{1-0.500} &= \frac{0.321}{0.500}=0.0.642,\\
\frac{0.424-0.251}{1-0.251} &= \frac{0.173}{0.749}=0.231,\\
\frac{0.682-0.436}{1-0.436} &= \frac{0.246}{0.564}=0.436.
\end{aligned}$$

Disregarding S_1, we see that the fuzzy similarity measures range from medium to high for the GII rank and the HDI rank.

Here $n=37$. Thus we obtain

$$
\begin{aligned}
S(\mu_A,\mu_C) &= 1-\frac{247}{(37)(38)}=0.824,\\
M(\mu_A,\mu_C) &= \frac{0.824}{2-0.824}=0.701,\\
S_L(\mu_A,\mu_C) &= 0.824+\frac{1}{37}(0.824-1)=0.819,\\
S_1(\mu_A,\mu_C) &= \frac{1}{37}(\frac{0.580}{37})=0.424,\\
S_2(\mu_A,\mu_C) &= \frac{1}{37}(25.14)=0.679.
\end{aligned}
$$

The smallest of these fuzzy similarity measures can be is determined immediately above.

$$
\begin{aligned}
\frac{0.824-0.514}{1-0.514} &= \frac{0.310}{0.486}=0.638,\\
\frac{0.701-0.345}{1-0.345} &= \frac{0.356}{0.655}=0.544,\\
\frac{0.819-0.500}{1-0.500} &= \frac{0.319}{0.500}=0.638,\\
\frac{0.424-0.251}{1-0.251} &= \frac{0.173}{0.749}=0.231,\\
\frac{0.679-0.436}{1-0.436} &= \frac{0.243}{0.564}=0.431.
\end{aligned}
$$

Disregarding S_1, we find that the fuzzy similarity measures range from medium to high for the Peace and Security rank and the GII rank.

5 Conclusions

Evidence collected at country and regional levels confirms high prevalence rates of violence against women and girls. The Women, Peace, and Security (WPS) Index ranks 177 countries and economies on women's status, [16]. Countries are also ranked according to their achievement of the rights of a child, [15]. The Gender Inequality Index (GII) ranks countries with respect to the loss of achievement within a country due to gender inequality, [14]. The Human Development Index (GHI) with respect to human development. In this paper, we determine the similarity of the rankings using various fuzzy similarity measures. We found that fuzzy similarity measures of these rankings ranged from medium to high depending on the particular measure used.

We considered the countries making up the Arab States. In future research, we will consider countries making up other regions.

Declarations

The author declares that they have no conflict of interest.

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