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Interval valued (α, β)-fuzzy hyperideals in Krasner (m, n)-hyperrings

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Abstract

In this paper, the notion of quasicoincidence of a fuzzy interval valued with an interval valued fuzzy set, which generalizes the concept of quasicoincidence of a fuzzy point in a fuzzy set is concentrated. Based on the idea, we study the concept of interval valued (α, β)-fuzzy hyperideals in Krasner (m, n)-hyperrings. In particular, some fundamental aspects of interval valued $(\in, \in \vee q)$-fuzzy hyperideals will be considered. Moreover, we examine the notion of implication-based interval valued fuzzy hyperideals in a Krasner (m, n)-hyperring.

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1 Introduction

In this section, we describe the motivation and a survey of related works. The concept of fuzzy sets was proposed by Zadeh in 1965 [26]. Fuzzy set theory is a useful tool to describe situations in which the data are imprecise or vague. After the introduction of the notion of hypergroups as a generalization of groups by Marty [19] in 1934, many papers and books concerning hyperstructure theory have appeared in literature [1, 5, 7, 8, 9, 10, 15, 16, 17, 23]. One important class of hyperrings is called the Krasner hyperring [14]. In [20], a generalization of the Krasner hyperrings, which is a subclass of (m, n)-hyperrings, was defined by Mirvakili and Davvaz. It is called Krasner (m, n)-hyperring. Many other interesting papers have been written on Krasner (m, n)-hyperring [2, 3, 4, 13, 20, 21, 22, 24].

Many researchers are interested in fuzzy hyperstructures because of nice connection between fuzzy sets and hyperstructures [6, 12, 18, 25, 28, 29, 32]. The notion of interval valued (α, β)-fuzzy subalgebraic hypersystems in an algebraic hypersystem, which is a generalization of a fuzzy subalgebraic system, was defined in [30]. Dutta in [11] established the concept of interval valued fuzzy prime and semiprime ideals of a hypersemiring.

In this paper, our aim is to consider the concept of quasicoincidence of a fuzzy interval valued with an interval valued fuzzy set, which generalizes the concept of quasicoincidence of a fuzzy point in a fuzzy set. This paper is organized as follows. In Section 2, we recall some terms and definitions which we need to develop our paper. In Section 3, we analyze entropy of interval valued (α,β)-fuzzy hyperideals in Krasner (m,n)-hyperrings. In Section 4, some fundamental aspects of interval valued $(\in,\in\vee q)$-fuzzy hyperideals have been investigated. Finally, in Section 5, we discuss the notion of implication-based interval valued fuzzy hyperideals of Krasner (m,n)-hyperrings.

2 Preliminaries

In this section, we recall some basic notions and results of fuzzy algebra and Krasner (m,n)-hyperrings which we shall use in this paper.

A fuzzy subset of G is a function $\mu : G \longrightarrow L$ such that L is the unit interval $[0,1] \subseteq \mathbb{R}$. The set of all fuzzy subsets of G is denoted by L^G . The set, $\{a \in G \mid \mu(a) \neq 0\}$ is called the support of μ and is denoted by $supp(\mu)$.

Definition 2.1. [27] *An interval number on* $[0,1]$*, denoted by* $\tilde{x}$*, is defined as the closed subinterval of* $[0,1]$*, where* $\tilde{x}=[x^-,x^+]$ *satisfying* $0 \leq x^- \leq x^+ \leq 1$.

$D[0,1]$ denotes the set of all interval numbers. The interval $[x,x]$ can be simply identified by the number x. Let $\tilde{x}_i=[x_i^-,x_i^+], \tilde{y}_i=[y_i^-,y_i^+] \in D[0,1]$ for $i \in I$. Then we define:

$$rmin\{\tilde{x}_i,\tilde{y}_i\}=[min\{x_i^-,y_i^-\},min\{x_i^+,y_i^+\}],$$

$$rmax\{\tilde{x}_i,\tilde{y}_i\}=[max\{x_i^-,y_i^-\},max\{x_i^+,y_i^+\}],$$
$$rsup\ \tilde{x}_i=[\bigvee_{i\in I} x_i^-,\bigvee_{i\in I} x_i^+],$$
$$rinf\ \tilde{x}_i=[\bigwedge_{i\in I} x_i^-,\bigwedge_{i\in I} x_i^+],$$

and put

(1) $x_1^- \leq x_2^-$ and $x_1^+ \leq x_2^+ \Longleftrightarrow \tilde{x}_1 \leq \tilde{x}_2$,
(2) $x_1^- = x_2^-$ and $x_1^+ = x_2^+ \Longleftrightarrow \tilde{x}_1 = \tilde{x}_2$,
(3) $\tilde{x}_1 \leq \tilde{x}_2$ and $\tilde{x}_1 \neq \tilde{x}_2 \Longleftrightarrow \tilde{x}_1 < \tilde{x}_2$,
(4) $k\tilde{x}=[kx^-,kx^+]$ for $0 \leq k \leq 1$.

Clearly, $(D[0,1],\leq,\wedge,\vee)$ forms a complete lattice with the least element $0=[0,0]$ and the greatest element $1=[1,1]$.
Recall from [27] that an interval valued fuzzy subset $\mathcal{A}$ on X is the set

$$\mathcal{A}=\{(x,[\tilde{\mu}_{\mathcal{A}}^-(x),\tilde{\mu}_{\mathcal{A}}^+(x)])|x \in X\},$$

such that $\tilde{\mu}_{\mathcal{A}}^-$ and $\tilde{\mu}_{\mathcal{A}}^+$ are two fuzzy subsets of X with $\tilde{\mu}_{\mathcal{A}}^-(x) \leq \tilde{\mu}_{\mathcal{A}}^+(x)$ for all $x \in X$. We put $\tilde{\mu}_{\mathcal{A}}(x)=[\tilde{\mu}_{\mathcal{A}}^-(x),\tilde{\mu}_{\mathcal{A}}^+(x)]$. Then we have $\mathcal{A}=\{(x,\tilde{\mu}_{\mathcal{A}}(x))|x \in X\}$ such that $\tilde{\mu}_{\mathcal{A}} : X \longrightarrow D[0,1]$.

An interval valued fuzzy set $\mathcal{A}$ of a Krasner (m,n)-hyperring $\mathcal{R}$ of the form

$$\tilde{\mu}_{\mathcal{A}}(y)=\begin{cases} \tilde{s}(\neq [0,0]) & \text{if } y=x,\\ [0,0] & \text{otherwise.}\end{cases}$$

is called a fuzzy interval value with support x and interval value $\tilde{s}$ and is denoted by $F(x;\tilde{s})$. A fuzzy interval value $F(x;\tilde{s})$ is said to belong to (resp. be quasi-coincident with) an interval valued fuzzy set $\mathcal{A}$, written as $F(x;\tilde{s}) \in \mathcal{A}$ (resp. $F(x;\tilde{s})q\mathcal{A}$) if $\tilde{\mu}_{\mathcal{A}}(x) \geq \tilde{s}$ (resp. $\tilde{\mu}_{\mathcal{A}}(x)+\tilde{s} > [1,1]$). We write $F(x;\tilde{s}) \in \vee q$ (resp. $F(x;\tilde{s}) \in \wedge q$) $\mathcal{A}$ if $F(x;\tilde{s}) \in \mathcal{A}$ or (resp. and) $F(x;\tilde{s})q\mathcal{A}$. If $\in \vee q$ does not hold, then we write $\overline{\in \vee q}$.

Suppose that G is a nonempty set. $P^*(G)$ denotes the set of all the nonempty subsets of G. The map $f : G^n \longrightarrow P^*(G)$ is called an n-ary hyperoperation and the algebraic system (G, f) is called an n-ary hypergroupoid. For non-empty subsets $G_1, ..., G_n$ of G we define $f(G_1^n) = f(G_1, ..., G_n) = \bigcup\{f(a_1^n) \mid a_i \in G_i, i = 1, ..., n\}$. The sequence $a_i, a_{i+1}, ..., a_j$ will be denoted by a_i^j. For $j < i$, a_i^j is the empty symbol. Using this notation, $f(a_1, ..., a_i, b_{i+1}, ..., b_j, c_{j+1}, ..., c_n)$ will be written as $f(a_1^i, b_{i+1}^j, c_{j+1}^n)$. The expression will be written in the form $f(a_1^i, b^{(j-i)}, c_{j+1}^n)$, when $b_{i+1} = ... = b_j = b$. If for every $1 \leq i < j \leq n$ and all $a_1, a_2, ..., a_{2n-1} \in G$,

$$f(a_1^{i-1}, f(a_i^{n+i-1}), a_{n+i}^{2n-1}) = f(a_1^{j-1}, f(a_j^{n+j-1}), a_{n+j}^{2n-1}),$$

then the n-ary hyperoperation f is called associative. An n-ary hypergroupoid with the associative n-ary hyperoperation is called an n-ary semihypergroup.

An n-ary hypergroupoid (G, f) in which the equation $y \in f(x_1^{i-1}, a_i, x_{i+1}^n)$ has a solution $a_i \in G$ for every $x_1^{i-1}, x_{i+1}^n, y \in G$ and $1 \leq i \leq n$, is called an n-ary quasihypergroup, when (G, f) is an n-ary semihypergroup, (G, f) is called an n-ary hypergroup.

An n-ary hypergroupoid (G, f) is commutative if for all $\sigma \in \mathbb{S}_n$, the group of all permutations of $\{1, 2, 3, ..., n\}$, and for every $x_1^n \in G$ we have $f(x_1, ..., x_n) = f(x_{\sigma(1)}, ..., x_{\sigma(n)})$. If $x_1^n \in G$ then we denote $x_{\sigma(1)}^{\sigma(n)}$ as the $(x_{\sigma(1)}, ..., x_{\sigma(n)})$.

Assume that f is an n-ary hyperoperation and $t = l(n-1)+1$, then t-ary hyperoperation $f_{(l)}$ is defined as $f_{(l)}(x_1^{l(n-1)+1}) = f(f(..., f(f(x_1^n), x_{n+1}^{2n-1}), ...), x_{(l-1)(n-1)+1}^{l(n-1)+1})$.

Definition 2.2. [20] *Let (G, f) be an n-ary hypergroup and H be a non-empty subset of G. H is an n-ary subhypergroup of (G, f), if $f(a_1^n) \subseteq H$ for $a_1^n \in H$, and the equation $b \in f(b_1^{i-1}, x_i, b_{i+1}^n)$ has a solution $x_i \in H$ for every $b_1^{i-1}, b_{i+1}^n, b \in H$ and $1 \leq i \leq n$. An element $e \in G$ is said to be a scalar neutral element if $a = f(e^{(i-1)}, a, e^{(n-i)})$, for every $1 \leq i \leq n$ and for every $a \in G$.*

An element 0 of an n-ary semihypergroup (G, g) is a zero element if $g(0, a_2^n) = g(a_2, 0, a_3^n) = ... = g(a_2^n, 0) = 0$ for all $a_2^n \in G$.

Definition 2.3. [15] *Let (G, f) be an n-ary hypergroup. (G, f) is called a canonical n-ary hypergroup if*
(1) *there exists a unique $e \in G$, such that $f(a, e^{(n-1)}) = a$ for all $a \in G$*
(2) *for all $a \in G$ there exists a unique $a^{-1} \in G$, such that $e \in f(a, a^{-1}, e^{(n-2)})$;*
(3) *if $a \in f(a_1^n)$, then for all i, we have $a_i \in f(a, a^{-1}, ..., a_{i-1}^{-1}, a_{i+1}^{-1}, ..., a_n^{-1})$.*

We say that e is the scalar identity of (G, f) and a^{-1} is the inverse of a. Notice that the inverse of e is e.

Definition 2.4. [20] *A Krasner (m, n)-hyperring is an algebraic hyperstructure (R, f, g), or simply R, which satisfies the following axioms:*

(1) *(R, f) is a canonical m-ary hypergroup;*

(2) *(R, g) is a n-ary semigroup;*

(3) *the n-ary operation g is distributive with respect to the m-ary hyperoperation f , i.e., for $x_1^{i-1}, x_{i+1}^n, a_1^m \in R$, and $1 \leq i \leq n$, $g(x_1^{i-1}, f(a_1^m), x_{i+1}^n) = f(g(x_1^{i-1}, a_1, x_{i+1}^n), ..., g(x_1^{i-1}, a_m, x_{i+1}^n))$;*

(4) *0 is a zero element of the n-ary operation g, i.e., for $a_2^n \in R$, $g(0, a_2^n) = g(a_2, 0, a_3^n) = ... = g(a_2^n, 0) = 0$.*

A non-empty subset S of R is called a subhyperring of R if (S, f, g) is a Krasner (m, n)-hyperring. Let I be a non-empty subset of R, we say that I is a hyperideal of (R, f, g) if (I, f) is an m-ary subhypergroup of (R, f) and $g(a_1^{i-1}, I, a_{i+1}^n) \subseteq I$, for every $a_1^n \in R$ and $1 \leq i \leq n$.

Definition 2.5. [8] *A Fuzzy subset $\mathcal{A}$ of a Krasner (m,n)-hyperring $\mathcal{R}$ is said to be a fuzzy hyperideal of $\mathcal{R}$ if the following conditions hold:*
(i) $min\{\mathcal{A}(a_1),\ldots,\mathcal{A}(a_m)\} \leq inf\{f(c)|\ c \in f(a_1^m)\}$ *for all* $a_1^m \in \mathcal{R}$*;*
(ii) $\mathcal{A}(a) \leq \mathcal{A}(-a)$*, for all* $a \in \mathcal{R}$*;*
(iii) $max\{\mathcal{A}(a_1),\ldots,\mathcal{A}(a_n)\} \leq \mathcal{A}(g(a_1^n))$*, for all* $a_1^n \in \mathcal{R}$*.*

It was shown (Theorem 5.6 in [8]) that a fuzzy subset $\mathcal{A}$ of a Krasner (m,n)-hyperring $\mathcal{R}$ is a fuzzy hyperideal if and only if every its non-empty level subset is a hyperideal of $\mathcal{R}$.

3 Interval valued (α,β)-fuzzy hyperideals

In this section, we introduce the notion of n-ary interval valued (α,β)-fuzzy hyperideal in a Krasner (m,n)-hyperring $\mathcal{R}$ where $\alpha,\beta \in \{\in, q, \in \vee q, \in \wedge q\}$.

Definition 3.1. *An interval valued fuzzy set $\mathcal{A}$ of a Krasner (m,n)-hyperring $\mathcal{R}$ is said to be n-ary interval valued (α,β)-fuzzy hyperideal of $\mathcal{R}$ where $\alpha,\beta \in \{\in, q, \in \vee q, \in \wedge q\}$ if for all $s_1^m, t_1^n, s \in (0,1]$ and $a_1^m, b_1^n, b \in \mathcal{R}$, the following conditions hold:*

(1) $F(a_1;\tilde{s}_1)\alpha\mathcal{A},\ldots,F(a_m;\tilde{s}_m)\alpha\mathcal{A}$ *impliy that* $F(a; rmin\{\tilde{s}_1,\ldots,\tilde{s}_m\})\beta\mathcal{A}$*, for all* $a \in f(a_1^m)$*,*

(2) $F(b;\tilde{s})\alpha\mathcal{A}$ *implies that* $F(-b;\tilde{s})\beta\mathcal{A}$*,*

(3) $F(b_1;\tilde{s}_1)\alpha\mathcal{A},\ldots,F(b_n;\tilde{s}_m)\alpha\mathcal{A}$ *imply that* $F(g(b_1^n);\tilde{s})\beta\mathcal{A}$*,*

Notice that $\alpha =\in \wedge q$ in Definition 3.1 should not be considered. Let $\tilde{\mu}_{\mathcal{A}}(a) \leq [0.5, 0.5]$ for an interval valued fuzzy set $\mathcal{A}$ of $\mathcal{R}$ and for all $a \in \mathcal{R}$. Assume that $F(a;\tilde{s}) \in \wedge q\mathcal{A}$ for $a \in \mathcal{R}$ and $s \in (0,1]$. This means $\tilde{\mu}_{\mathcal{A}}(a) \geq \tilde{s}$ and $\tilde{\mu}_{\mathcal{A}}(a) + \tilde{s} > [1,1]$. Then we have

$$[1,1] < \tilde{\mu}_{\mathcal{A}}(a) + \tilde{s} \leq \tilde{\mu}_{\mathcal{A}}(a) + \tilde{\mu}_{\mathcal{A}}(a) = 2\tilde{\mu}_{\mathcal{A}}(a),$$

and so $\tilde{\mu}_{\mathcal{A}}(a) > [0.5, 0.5]$. This follows that $\{F(a;\tilde{s}) \mid F(a;\tilde{s}) \in \wedge q\mathcal{A}\} = \varnothing$.

Example 3.2. *Consider the $(2,2)$-hyperring $(R = \{x,y,z,w\}, \boxplus, \cdot)$ such that the hyperoperation "$\boxplus$" and operation "$\cdot$" are defined as:*

$\boxplus$	x	y	z	w
x	$\{x\}$	$\{y\}$	$\{z\}$	$\{w\}$
y	$\{y\}$	$\{x,y\}$	$\{w\}$	$\{z\}$
z	$\{z\}$	$\{w\}$	$\{x,z\}$	$\{y\}$
w	$\{w\}$	$\{z\}$	$\{y\}$	$\{x,w\}$

$\cdot$	x	y	z	w
x	x	x	x	x
y	x	y	y	y
z	x	z	z	z
w	x	w	w	w

Let

$$\tilde{\mu}_{\mathcal{A}}(a) = \begin{cases} [0.2, 0.3] & \text{if } a = x, \\ [0.6, 0.7] & \text{if } a = y, z, w. \end{cases}$$

Then it is easy to check that $\mathcal{A}$ is a 2-ary interval valued $(\in, \in \wedge q)$-fuzzy hyperideal of $\mathcal{R}$.

Theorem 3.3. *Let $\mathcal{A}$ be an n-ary interval valued $(\in,\in)$-fuzzy hyperideal of a Krasner (m,n)-hyperring $\mathcal{R}$. Then $\mathcal{A}$ is an n-ary interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $\mathcal{R}$.*

Proof. The proof is straightforward. □

Theorem 3.4. *Let $\mathcal{A}$ be an n-ary interval valued $(\in \vee q, \in \vee q)$-fuzzy hyperideal of a Krasner (m, n)-hyperring $\mathcal{R}$. Then $\mathcal{A}$ is an n-ary interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $\mathcal{R}$.*

Proof. Let $\mathcal{A}$ be an n-ary interval valued $(\in \vee q, \in \vee q)$-fuzzy hyperideal of $\mathcal{R}$. Let $F(a_1; \tilde{s}_1) \in \mathcal{A}, \ldots, F(a_m; \tilde{s}_m) \in \mathcal{A}$ for all $s_1^m \in (0, 1]$ and $a_1^m \in \mathcal{R}$. Therefore $F(a_1; \tilde{s}_1) \in \vee q\mathcal{A}, \ldots, F(a_m; \tilde{s}_m) \in \vee q\mathcal{A}$. Since $\mathcal{A}$ be an n-ary interval valued $(\in \vee q, \in \vee q)$-fuzzy hyperideal of $\mathcal{R}$, then for all $a \in f(a_1^m)$, $F(a; rmin\{\tilde{s}_1, \ldots, \tilde{s}_m\}) \in \vee q\mathcal{A}$. The proofs of the other cases are similar. □

Theorem 3.5. *Let I be a hyperideal of a Krasner (m, n)-hyperring $\mathcal{R}$. Then the characteristic function χ_I of I is an n-ary interval valued $(\in, \in)$-fuzzy hyperideal of $\mathcal{R}$.*

Proof. Let I be a hyperideal of a Krasner (m, n)-hyperring $\mathcal{R}$. Let $F(a_1; \tilde{s}_1) \in \chi_I, \ldots, F(a_m; \tilde{s}_m) \in \chi_I$ for all $s_1^m \in (0, 1]$ and $a_1^m \in \mathcal{R}$. Then for $1 \leq i \leq m$, $\tilde{\chi}_I(a_i) \geq \tilde{s}_i > [0, 0]$. Then we get $\tilde{\chi}_I(a_i) = [1, 1]$ for all $1 \leq i \leq m$. This means $a_i \in \chi_I$ for all $1 \leq i \leq m$. Therefore $a \in \chi_I$ for all $a \in f(a_1^m)$. Thus $\tilde{\chi}_I(a) = [1, 1] \geq rmin\{\tilde{s}_1, \ldots, \tilde{s}_m\}$ which implies $F(a; rmin\{\tilde{s}_1, \ldots, \tilde{s}_m\}) \in \chi_I$. The proofs of the other conditions are similar. □

Let $\mathcal{A}$ be an interval valued fuzzy set. The set $F(\mathcal{A}; \tilde{s}) = \{a \in \mathcal{R} \mid \tilde{\mu}_{\mathcal{A}}(a) \geq \tilde{s}\}$ is called the interval valued level subset of $\mathcal{A}$. We say that an interval valued fuzzy set $\mathcal{A}$ of a Krasner (m, n)-hyperring $\mathcal{R}$ is proper if $|Im\mathcal{A}| \geq 2$. If two interval valued fuzzy sets have the same family of interval valued level subsets, then they are said to be equivalent.

Theorem 3.6. *Let $\mathcal{R}$ be a Krasner (m, n)-hyperring containing some proper hyperideals and let $\mathcal{A}$ be an proper interval valued $(\in, \in)$-fuzzy hyperideal of $\mathcal{R}$ with $|Im\mathcal{A}| \geq 3$. Then $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$ such that $\mathcal{A}_1$ and $\mathcal{A}_2$ are non-equivalent interval valued $(\in, \in)$-fuzzy hyperideals of $\mathcal{R}$.*

Proof. The proof is similar to the proof of Theorem 3.7 in [31]. □

4 n-ary interval valued $(\in, \in \vee q)$-fuzzy hyperideals

In this section, we first generalize the notion of fuzzy hyperideals to the notion of interval valued fuzzy hyperideals in a Krasner (m, n)-hyperring. Then we study some fundamental aspects of the interval valued $(\in, \in \vee q)$-fuzzy hyperideals in a Krasner (m, n)-hyperring.

Definition 4.1. *Let $\mathcal{A}$ be an interval valued fuzzy set of a Krasner (m, n)-hyperring $\mathcal{R}$. $\mathcal{A}$ refers to an n-ary interval valued fuzzy hyperideal if for all $a_1^m, b_1^n, b \in \mathcal{R}$, the following conditions hold:*

(i) $rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m)\} \leq rinf\{\tilde{\mu}_{\mathcal{A}}(a) \mid a \in f(a_1^m)\}$,

(ii) $\tilde{\mu}_{\mathcal{A}}(b) \leq \tilde{\mu}_{\mathcal{A}}(-b)$

(iii) $rmax\{\tilde{\mu}_{\mathcal{A}}(b_1), \ldots, \tilde{\mu}_{\mathcal{A}}(b_n)\} \leq \tilde{\mu}_{\mathcal{A}}(g(b_1^n))$.

The following is a direct consequence and can be proved easily and so the proof is omited.

Theorem 4.2. *Let $\mathcal{A}$ be an interval valued fuzzy set of a Krasner (m, n)-hyperring $\mathcal{R}$. Then $\mathcal{A}$ is an n-ary interval valued fuzzy hyperideal of $\mathcal{R}$ if and only if for each $[0, 0] < \tilde{s} \leq [1, 1]$, $F(\mathcal{A}; \tilde{s})(\neq \varnothing)$ is a hyperideal of $\mathcal{R}$.*

Proof. $\Longrightarrow$ Let $\mathcal{A}$ be an n-ary interval valued fuzzy hyperideal of $\mathcal{R}$ and $a_1^m \in F(\mathcal{A};\tilde{s})$. Then we get $rmin\{\tilde{\mu}_{\mathcal{A}}(a_1),\ldots,\tilde{\mu}_{\mathcal{A}}(a_m)\} \geq \tilde{s}$. Therefore $rinf\{\tilde{\mu}_{\mathcal{A}}(a) \mid a \in f(a_1^m)\} \geq \tilde{s}$ which means $a \in F(\mathcal{A};\tilde{s})$ for all $a \in f(a_1^m)$. Thus $f(a_1^m) \subseteq F(\mathcal{A};\tilde{s})$. Also, since $\tilde{\mu}_{\mathcal{A}}(b) \leq \tilde{\mu}_{\mathcal{A}}(-b)$, we have $-b \in F(\mathcal{A};\tilde{s})$ for $b \in F(\mathcal{A};\tilde{s})$. Now, assume that $b_1^n \in \mathcal{R}$ and $b_i \in F(\mathcal{A};\tilde{s})$ for some $1 \leq i \leq n$. Therefore $\tilde{\mu}_{\mathcal{A}}(b_i) \geq \tilde{s}$. Hence $\tilde{s} \leq rmax\{\tilde{\mu}_{\mathcal{A}}(b_1),\ldots,\tilde{\mu}_{\mathcal{A}}(b_n)\} \leq \tilde{\mu}_{\mathcal{A}}(g(b_1^n))$. Thus $g(b_1^{i-1}, F(\mathcal{A};\tilde{s}), b_{i+1}^n) \subseteq F(\mathcal{A};\tilde{s})$. Consequently, $F(\mathcal{A};\tilde{s})$ is a hyperideal of $\mathcal{R}$.

$\Longleftarrow$ Since $\tilde{\mu}_{\mathcal{A}}(a_i) \geq rmin\{\tilde{\mu}_{\mathcal{A}}(a_1),\ldots,\tilde{\mu}_{\mathcal{A}}(a_m)\} = \tilde{s}_0$ for all $a_1^m \in \mathcal{R}$, we get $a_i \in F(\mathcal{A};\tilde{s}_0)$. By the assumption, we have $f(a_1^m) \subseteq F(\mathcal{A};\tilde{s}_0)$ which implies $\tilde{\mu}_{\mathcal{A}}(a) \geq \tilde{s}_0$ for all $a \in f(a_1^m)$. Therefore we conclude that $\tilde{s} \leq rmin\{\tilde{\mu}_{\mathcal{A}}(a_1),\ldots,\tilde{\mu}_{\mathcal{A}}(a_m)\} \leq rinf\{\tilde{\mu}_{\mathcal{A}}(a) \mid a \in f(a_1^m)\}$. The other conditions can be proved easily. □

Definition 4.3. *Let $\mathcal{A}$ be an interval valued fuzzy set of a Krasner (m,n)-hyperring $\mathcal{R}$. $\mathcal{A}$ is called an n-ary interval valued $(\in,\in\vee q)$-fuzzy hyperideal of $\mathcal{R}$ if for all $s_1^m, t_1^n, t \in (0,1]$ and $a_1^m, b_1^n, b \in \mathcal{R}$*

(i1) $F(a_1;\tilde{s}_1) \in \mathcal{A},\ldots,F(a_m;\tilde{s}_m) \in \mathcal{A}$ *imply* $F(a; rmin\{\tilde{s}_1,\ldots,\tilde{s}_m\}) \in \vee q\mathcal{A}$, *for all* $a \in f(a_1^m)$,

(ii1) $F(b;\tilde{t}) \in \mathcal{A}$ *implies* $F(-b;\tilde{t}) \in \vee q\mathcal{A}$

(iii1) $F(b_1;\tilde{t}_1) \in \mathcal{A},\ldots,F(b_n;\tilde{t}_n) \in \mathcal{A}$ *imply* $F(g(b_1^n); rmax\{\tilde{t}_1,\ldots,\tilde{t}_n\}) \in \vee q\mathcal{A}$

It is clear that every n-ary interval valued fuzzy hyperideal of a Krasner (m,n)-hyperring is an n-ary interval valued $(\in,\in\vee q)$-fuzzy hyperideal of $\mathcal{R}$. The following example shows that the inverse is not true, in general.

Example 4.4. *The set $R=\{0,1,2,3\}$ with following 2-hyperoperation is a canonical 2-ary hypergroup:*

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	A	3	B
2	2	3	0	1
3	3	B	1	A

in which $A=\{0,1\}$ and $B=\{2,3\}$. Define a 4-ary operation g on R as follows:

$$g(a_1^4) = \begin{cases} 2 & \text{if } a_1,a_2,a_3,a_4 \in B \\ 0 & \text{otherwise.} \end{cases}$$

It follows that $(R,\oplus,g)$ is a Krasner (2,4)-hyperring. Now, we define $\tilde{\mu}_{\mathcal{A}}(0)=\tilde{\mu}_{\mathcal{A}}(1)=[0.8,0.9]$, $\tilde{\mu}_{\mathcal{A}}(2)=[0.7,0.8]$ and $\tilde{\mu}_{\mathcal{A}}(3)=[0.6,0.7]$. Then it is easy to check that $\mathcal{A}$ is an interval valued $(\in,\in\vee,q)$-)-fuzzy hyperideal of $\mathcal{R}$.

In the following theorem, we present an equivalent condition for n-ary interval valued $(\in,\in\vee q)$-fuzzy hyperideals.

Theorem 4.5. *Let $\mathcal{A}$ be an interval valued fuzzy set of a Krasner (m,n)-hyperring $\mathcal{R}$. Then $\mathcal{A}$ is an n-ary interval valued $(\in,\in\vee q)$-fuzzy hyperideal of $\mathcal{R}$ if and only if for all $a_1^m, b_1^n, b \in \mathcal{R}$,*

(i2) $rmin\{\tilde{\mu}_{\mathcal{A}}(a_1),\ldots,\tilde{\mu}_{\mathcal{A}}(a_m),[0.5,0.5]\} \leq rinf\{\tilde{\mu}_{\mathcal{A}}(a) \mid a \in f(a_1^m)\}$,

(ii2) $rmin\{\tilde{\mu}_{\mathcal{A}}(b),[0.5.0.5]\} \leq \tilde{\mu}_{\mathcal{A}}(-b)$,

(iii2) $rmax\{\tilde{\mu}_{\mathcal{A}}(b_1),\ldots,\tilde{\mu}_{\mathcal{A}}(b_n),[0.5,0.5]\} \leq \tilde{\mu}_{\mathcal{A}}(g(b_1^n))$.

Proof. We need to show that the conditions of Definition 4.3 are equivalent to the conditions (i2), (ii2) and (iii2), respectively.

$(i1) \Longrightarrow (i2)$: Let $a_1^m \in \mathcal{R}$. Then There exist two cases to be considered:

(1) $rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m)\} \leq [0.5, 0.5]$,

(2) $rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m)\} > [0.5, 0.5]$.

Case (1): Let $\tilde{\mu}_{\mathcal{A}}(a) < rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m), [0.5, 0.5]\}$, for some $a \in f(a_1^m)$. Therefore $\tilde{\mu}_{\mathcal{A}}(a) < rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m)\}$. Take $s \in (0, 1]$ that $\tilde{\mu}_{\mathcal{A}}(a) < \tilde{s} < rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m)\}$. This implies that for all $1 \leq i \leq m$, $F(a_i; \tilde{s}) \in \mathcal{A}$ but $F(a; \tilde{s}) \overline{\in \vee q} \mathcal{A}$, a contradiction.

Case (2): Suppose that there exists $a \in f(a_1^m)$ with $\tilde{\mu}_{\mathcal{A}}(a) < [0.5, 0.5]$. Then we conclude that $F(a_i; [0.5, 0.5]) \in \mathcal{A}$ for all $1 \leq i \leq m$ as $rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m)\} > [0.5, 0.5]$. In the other hand, $F(a; [0.5.0.5]) \overline{\in \vee q} \mathcal{A}$, a contradiction.

$(ii1) \Longrightarrow (ii2)$: Let $b \in \mathcal{R}$. Then we have the following two cases:

(1) $\tilde{\mu}_{\mathcal{A}}(b) \leq [0.5, 0.5]$,

(2) $\tilde{\mu}_{\mathcal{A}}(b) > [0.5, 0.5]$.

Case (1): Let us consider $\tilde{\mu}_{\mathcal{A}}(b) = \tilde{s} < [0.5, 0.5]$ and $\tilde{\mu}_{\mathcal{A}}(-b) = \tilde{t} < \tilde{\mu}_{\mathcal{A}}(b)$. We take r such that $\tilde{t} < \tilde{r} < \tilde{s}$ and $\tilde{t} + \tilde{r} < [0.5, 0, 5]$. Thus we get $F(b; \tilde{r}) \in \mathcal{A}$ but $F(-b; \tilde{r}) \overline{\in \vee q} \mathcal{A}$. This is a contradiction.

Case (2): Suppose that $\tilde{\mu}_{\mathcal{A}}(b) \geq [0.5, 0.5]$ and $rmin\{\tilde{\mu}_{\mathcal{A}}(b), [0.5.0.5]\} > \tilde{\mu}_{\mathcal{A}}(-b)$. Therefore we obtain $F(b; [0.5, 0.5]) \in \mathcal{A}$ but $F(-b; [0.5, 0.5]) \overline{\in \vee q} \mathcal{A}$, a contradiction.

$(iii1) \Longrightarrow (iii2)$ By using an argument similar to that in the proof of $(i1) \Longrightarrow (i2)$ one can easily complete the proof.

$(i2) \Longrightarrow (i1)$ Let $F(a_1; \tilde{s}_1) \in \mathcal{A}, \ldots, F(a_m; \tilde{s}_m) \in \mathcal{A}$ for $a_1^m \in \mathcal{R}$ and $s_1^m \in (0, 1]$. This means that for all $1 \leq i \leq m$ we get $\tilde{\mu}_{\mathcal{A}}(a_i) \geq \tilde{s}_i$. On the other hand, we have $rmin\{\tilde{s}_1, \ldots, \tilde{s}_m, [0.5, 0.5]\} \leq rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m), [0.5, 0.5]\} \leq \tilde{\mu}_{\mathcal{A}}(a)$ for each $a \in f(a_1^m)$. Now, if $rmin\{\tilde{s}_1, \ldots, \tilde{s}_m\} > [0.5, 0.5]$, then we have $\tilde{\mu}_{\mathcal{A}}(a) \geq [0.5, 0.5]$ and so $\tilde{\mu}_{\mathcal{A}}(a) + rmin\{\tilde{s}_1, \ldots, \tilde{s}_m\} > [1, 1]$. Otherwise, we get $\tilde{\mu}_{\mathcal{A}}(a) \geq rmin\{\tilde{s}_1, \ldots, \tilde{s}_m\}$ and then $F(a; rmin\{\tilde{s}_1, \ldots, \tilde{s}_m\}) \in \vee q \mathcal{A}$ for each $a \in f(a_1^m)$.

$(ii2) \Longrightarrow (ii1)$ Suppose that $F(b; \tilde{s}) \in \mathcal{A}$ for some $b \in \mathcal{R}$ and $s \in (0, 1]$. This means $\tilde{\mu}_{\mathcal{A}}(b) \geq \tilde{s}$. Thus, $rmin\{\tilde{s}, [0.5, 0.5])\} \leq rmin\{\tilde{\mu}_{\mathcal{A}}(b), [0.5, 0.5])\} \leq \tilde{\mu}_{\mathcal{A}}(-b)$. Now, if $\tilde{s} \leq [0.5, 0.5]$, then $\tilde{\mu}_{\mathcal{A}}(-b) \geq \tilde{s}$ and if $\tilde{s} \geq [0.5, 0.5]$, then we get $\tilde{\mu}_{\mathcal{A}}(-b) \geq [0.5, 0.5]$.

$(iii2) \Longrightarrow (iii1)$ This can be proved in a very similar manner to the way in which $(i2) \Longrightarrow (i1)$ was proved. □

Theorem 4.6. *Let I be a subset of a Krasner (m, n)-hyperring $\mathcal{R}$. I is a hyperideal of $\mathcal{R}$ if and only if χ_I is an n-ary interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $\mathcal{R}$.*

Proof. $\Longrightarrow$ Let I be a hyperideal of $\mathcal{R}$. By Theorem 3.5, we conclude that χ_I is an n-ary interval valued $(\in, \in)$-fuzzy hyperideal of $\mathcal{R}$. Thus χ_I is an n-ary interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $\mathcal{R}$, by Theorem 3.3.

$\Longleftarrow$ Suppose that χ_I is an n-ary interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $\mathcal{R}$. Let $a_1^m \in I$. This means $F(a_i; [1, 1]) \in \chi_I$ for all $1 \leq i \leq m$. Thus $F(a; [1, 1]) = F(a; rmin\{\underbrace{[1, 1], \ldots, [1, 1]}_{m}\}) \in \vee q \chi_I$, for all $a \in f(a_1^m)$. Therefore $\tilde{\mu}_I(a) > [0, 0]$ for all $a \in f(a_1^m)$ which implies $f(a_1^m) \subseteq I$. Assume that $a \in I$. Therefore $F(a; [1, 1]) \in \chi_I$ which means $F(-a; [1, 1]) \in \vee q \chi_I$. Hence $\tilde{\chi}_I(-a) > [0, 0]$. Consequently, we get $-a \in I$. Now we suppose that $b_1^n \in \mathcal{R}$ and $b_i \in I$ for some $1 \leq i \leq n$. Then we get $F(b_i;[1,1]) \in \chi_I$. From $rmax\{\tilde{\chi}_I(b_1),\ldots,\tilde{\chi}_I(b_{i-1}),\tilde{\chi}_I(b_i),\tilde{\chi}_I(b_{i+1}),\ldots,\tilde{\chi}_I(b_n),[0.5,0.5]\} \leq \tilde{\chi}_I(g(b_1^n))$, it follows that $F(g(b_1^n);[1,1]) \in \chi_I$. Thus $g(b_1^{i-1},b_i,b_{i+1}^n) \in I$. Hence I is a hyperideal of $\mathcal{R}$. □

In the following theorem, we examine n-ary interval valued $(\in,\in\vee q)$-fuzzy hyperideals by level subsets.

Theorem 4.7. *If $\mathcal{A}$ is an n-ary interval valued $(\in,\in\vee q)$-fuzzy hyperideal of a Krasner (m,n)-hyperring $\mathcal{R}$, then $F(\mathcal{A};\tilde{s})$ is an empty set or a hyperideal of $\mathcal{R}$ for every $[0,0] < \tilde{s} \leq [0.5,0.5]$.*

Proof. Suppose that $\mathcal{A}$ is an n-ary interval valued $(\in,\in\vee q)$-fuzzy hyperideal of $\mathcal{R}$ and $[0,0] < \tilde{s} \leq [0.5,0.5]$. Let $a_1^m \in F(\mathcal{A};\tilde{s})$. This means that $\tilde{\mu}_{\mathcal{A}}(a_i) \geq \tilde{s}$ for all $1 \leq i \leq m$ and so $\tilde{s} = rmin\{\tilde{s},[0.5,0.5]\} \leq rmin\{\tilde{\mu}_{\mathcal{A}}(a_1),\ldots,\tilde{\mu}_{\mathcal{A}}(a_m),[0.5,0.5]\} \leq rinf\{\tilde{\mu}_{\mathcal{A}}(a) \mid a \in f(a_1^m)\}$. Thus $f(a_1^m) \subseteq F(\mathcal{A};\tilde{s})$. Let $b \in F(\mathcal{A};\tilde{s})$. Then $\tilde{\mu}_{\mathcal{A}}(b) \geq \tilde{s}$. Now we get $\tilde{\mu}_{\mathcal{A}}(-b) \geq rmin\{\tilde{\mu}_{\mathcal{A}}(b),[0.5,0.5]\} = rmin\{\tilde{s},[0.5,0.5]\} = \tilde{s}$ which implies $-b \in F(\mathcal{A};\tilde{s})$. Moreover, let $b_1^n \in \mathcal{R}$ such that $b_i \in F(\mathcal{A};\tilde{s})$. Then $\tilde{\mu}_{\mathcal{A}}(g(b_1^n)) \geq rmax\{\tilde{\mu}_{\mathcal{A}}(b_1),\ldots,\tilde{\mu}_{\mathcal{A}}(b_{i-1}),\tilde{\mu}_{\mathcal{A}}(b_i),\tilde{\mu}_{\mathcal{A}}(b_{i+1}),\ldots,\tilde{\mu}_{\mathcal{A}}(b_n),[0.5,0.5]\} \geq \tilde{s}$. Hence we conclude that $g(b_1^n) \in F(\mathcal{A};\tilde{s})$. Consequently, $F(\mathcal{A};\tilde{s})$ is a hyperideal of $\mathcal{R}$. □

In the next theorem, the subsets are discussed where $[0.5,0.5] < \tilde{s} \leq [1,1]$.

Theorem 4.8. *Let $\mathcal{A}$ be an interval valued fuzzy set of a Krasner (m,n)-hyperring $\mathcal{R}$. Then $F(\mathcal{A};\tilde{s})(\neq\varnothing)$ is a hyperideal of $\mathcal{R}$ for every $[0.5,0.5] < \tilde{s} \leq [1,1]$ if and only if for all $a_1^m, b_1^n, b \in \mathcal{R}$,*

(1) $rmin\{\tilde{\mu}_{\mathcal{A}}(a_1),\ldots,\tilde{\mu}_{\mathcal{A}}(a_m)\} \leq rinf\{rmax\{\tilde{\mu}_{\mathcal{A}}(a),[0.5,0.5]\} \mid a \in f(a_1^m)\}$,

(2) $\tilde{\mu}_{\mathcal{A}}(b) \leq rmax\{\tilde{\mu}_{\mathcal{A}}(-b),[0.5.0.5]\}$,

(3) $rmax\{\tilde{\mu}_{\mathcal{A}}(b_1),\ldots,\tilde{\mu}_{\mathcal{A}}(b_n)\} \leq rmax\{\tilde{\mu}_{\mathcal{A}}(g(b_1^n)),[0.5,0.5]\}$.

Proof. $\Longrightarrow$ Suppose that $F(\mathcal{A};\tilde{s})(\neq\varnothing)$ is a hyperideal of $\mathcal{R}$ for every $[0.5,0.5] < \tilde{s} \leq [1,1]$.
(1) Let $rmax\{\tilde{\mu}_{\mathcal{A}}(a),[0.5,0.5]\} < rmin\{\tilde{\mu}_{\mathcal{A}}(a_1),\ldots,\tilde{\mu}_{\mathcal{A}}(a_m)\} = \tilde{s}$ for some $a_1^m \in \mathcal{R}$ such that $a \in f(a_1^m)$. This implies that $[0.5,0.5] < \tilde{s} \leq [1,1]$ and $a_1^m \in F(\mathcal{A};\tilde{s})$. Since $F(\mathcal{A};\tilde{s})$ is a hyperideal of $\mathcal{R}$ and $a_1^m \in F(\mathcal{A};\tilde{s})$, we get $f(a_1^m) \subseteq F(\mathcal{A};\tilde{s})$. This means that $\tilde{\mu}_{\mathcal{A}}(a) \geq \tilde{s}$ for each $a \in f(a_1^m)$, a contradiction. Thus $rmin\{\tilde{\mu}_{\mathcal{A}}(a_1),\ldots,\tilde{\mu}_{\mathcal{A}}(a_m)\} \leq rinf\{rmax\{\tilde{\mu}_{\mathcal{A}}(a),[0.5,0.5]\} \mid a \in f(a_1^m)\}$ for all $a_1^m \in \mathcal{R}$.
(2) Suppose that $\tilde{s} = \tilde{\mu}_{\mathcal{A}}(b) \geq rmax\{\tilde{\mu}_{\mathcal{A}}(-b),[0.5.0.5]\}$, for some $b \in \mathcal{R}$. This implies that $[0.5,0.5] < \tilde{s} \leq [1,1]$ and $b \in F(\mathcal{A};\tilde{s})$. Since $F(\mathcal{A};\tilde{s})$ is a hyperideal of $\mathcal{R}$ and $b \in F(\mathcal{A};\tilde{s})$, we conclude that $-b \in F(\mathcal{A};\tilde{s})$ which means $\tilde{\mu}_{\mathcal{A}}(-b) \geq \tilde{s}$, a contradiction. Hence $\tilde{\mu}_{\mathcal{A}}(b) \leq rmax\{\tilde{\mu}_{\mathcal{A}}(-b),[0.5.0.5]\}$ for $b \in \mathcal{R}$.
(3) By a similar argument to that of (1), we can prove

$$rmax\{\tilde{\mu}_{\mathcal{A}}(b_1),\ldots,\tilde{\mu}_{\mathcal{A}}(b_n)\} \leq rmax\{\tilde{\mu}_{\mathcal{A}}(g(b_1^n)),[0.5,0.5]\},$$

for $b_1^n \in \mathcal{R}$.
$\Longleftarrow$ Let $a_1^m \in F(\mathcal{A};\tilde{s})$ with $[0.5,0.5] < \tilde{s} \leq [1,1]$. Then $[0.5,0.5] < \tilde{s} \leq rmin\{\tilde{\mu}_{\mathcal{A}}(a_1),\ldots,\tilde{\mu}_{\mathcal{A}}(a_m)\} \leq rinf\{rmax\{\tilde{\mu}_{\mathcal{A}}(a),[0.5,0.5]\} \mid a \in f(a_1^m)\}$ and so $\tilde{s} \leq rinf\{\tilde{\mu}_{\mathcal{A}}(a) \mid a \in f(a_1^m)\}$. Thus $f(a_1^m) \subseteq F(\mathcal{A};\tilde{s})$. Also, let $b \in F(\mathcal{A};\tilde{s})$. Then we get $[0.5,0.5] < \tilde{s} \leq \tilde{\mu}_{\mathcal{A}}(b) \leq rmax\{\tilde{\mu}_{\mathcal{A}}(-b),[0.5.0.5]\}$. Therefore $\tilde{s} \leq \tilde{\mu}_{\mathcal{A}}(-b)$ which means $-b \in F(\mathcal{A};\tilde{s})$. Now, let $b_1^n \in \mathcal{R}$ and $b_i \in F(\mathcal{A};\tilde{s})$. Then we obtain

$$\begin{aligned}[0.5,0.5] < \tilde{s} &\leq rmax\{\tilde{\mu}_{\mathcal{A}}(b_1),\ldots,\tilde{\mu}_{\mathcal{A}}(b_{i-1}),\tilde{\mu}_{\mathcal{A}}(b_i),\tilde{\mu}_{\mathcal{A}}(b_{i+1}),\ldots\tilde{\mu}_{\mathcal{A}}(b_n)\}\\ &\leq rmax\{\tilde{\mu}_{\mathcal{A}}(g(b_1^n)),[0.5,0.5]\}.\end{aligned}$$

Hence $\tilde{s} \leq \tilde{\mu}_{\mathcal{A}}(g(b_1^n))$ and so $g(b_1^n) \in F(\mathcal{A}; \tilde{s})$. Consequently, $F(\mathcal{A}; \tilde{s})$ is a hyperideal of $\mathcal{R}$ for every $[0.5, 0.5] < \tilde{s} \leq [1, 1]$. □

Definition 4.9. *Let $\mathcal{A}$ be an interval valued fuzzy set of a Krasner (m, n)-hyperring $\mathcal{R}$ and $s_1, s_2 \in [0, 1]$ with $\tilde{s}_1 < \tilde{s}_2$. $\mathcal{A}$ refers to an n-ary interval valued fuzzy hyperideal with thresholds $(\tilde{s}_1, \tilde{s}_2)$ of $\mathcal{R}$ if for all $a_1^m, b_1^n, b \in \mathcal{R}$, $\mathcal{A}$ satisfies the following conditions:*

(1) $rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m), \tilde{s}_2\} \leq rinf\{rmax\{\tilde{\mu}_{\mathcal{A}}(a), \tilde{s}_1\} \mid a \in f(a_1^m)\}$,

(2) $rmin\{\tilde{\mu}_{\mathcal{A}}(b), \tilde{s}_2\} \leq rmax\{\tilde{\mu}_{\mathcal{A}}(-b), \tilde{s}_1\}$,

(3) $rmax\{\tilde{\mu}_{\mathcal{A}}(b_1), \ldots, \tilde{\mu}_{\mathcal{A}}(b_n), \tilde{s}_2\} \leq rmax\{\tilde{\mu}_{\mathcal{A}}(g(b_1^n)), \tilde{s}_1\}$.

Suppose that $\mathcal{A}$ is an n-ary interval valued fuzzy hyperideal with thresholds of a Krasner (m, n)-hyperring $\mathcal{R}$. Let us consider $\tilde{s}_1 = [0, 0]$ and $\tilde{s}_2 = [1, 1]$. Then $\mathcal{A}$ is an ordinary interval valued fuzzy hyperideal. Moreover, if we consider $\tilde{s}_1 = [0, 0]$ and $\tilde{s}_2 = [0.5, 0.5]$, then $\mathcal{A}$ is an n-ary interval valued $(\in, \in \vee q)$-fuzzy hyperideal.

Theorem 4.10. *Let $\mathcal{A}$ be an interval valued fuzzy set of a Krasner (m, n)-hyperring $\mathcal{R}$. Then $\mathcal{A}$ is an n-ary interval valued fuzzy hyperideal with thresholds $(\tilde{s}_1, \tilde{s}_2)$ of $\mathcal{R}$ if and only if $F(\mathcal{A}; \tilde{s})$ is a hyperideal of $\mathcal{R}$ for every $\tilde{s}_1 < \tilde{s} \leq \tilde{s}_2$.*

Proof. $\Longrightarrow$ Let $a_1^m \in F(\mathcal{A}; \tilde{s})$. Thereby we have $\tilde{\mu}_{\mathcal{A}}(a_i) \geq \tilde{s}$ for all $1 \leq i \leq m$. Then we deduce $\tilde{s}_1 < \tilde{s} \leq rmin\{\tilde{s}, \tilde{s}_2\} \leq rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m), \tilde{s}_2\}$ and so $rinf\{rmax\{\tilde{\mu}_{\mathcal{A}}(a), \tilde{s}_1\} \mid a \in f(a_1^m)\} \geq \tilde{s}$. This means that $rmax\{\tilde{\mu}_{\mathcal{A}}(a), \tilde{s}_1\} > \tilde{s}$ for all $a \in f(a_1^m)$. Then we get $\tilde{\mu}_{\mathcal{A}}(a) > \tilde{s}$ which implies $a \in F(\mathcal{A}; \tilde{s})$. Therefore $f(a_1^m) \subseteq F(\mathcal{A}; \tilde{s})$. Also, we assume that $b \in F(\mathcal{A}; \tilde{s})$. Then $rmax\{\tilde{\mu}_{\mathcal{A}}(-b), \tilde{s}_1\} \geq rmin\{\tilde{\mu}_{\mathcal{A}}(b), \tilde{s}_2\} \geq \tilde{s} > \tilde{s}_1$. Thereby we have $\tilde{\mu}_{\mathcal{A}}(-b) \geq \tilde{s}$ which means $-b \in F(\mathcal{A}; \tilde{s})$. Now, we consider $b_1^n \in \mathcal{R}$ and $b_i \in F(\mathcal{A}; \tilde{s})$. Then $\tilde{\mu}_{\mathcal{A}}(b_i) \geq \tilde{s}$. Hence we obtain $rmax\{\tilde{\mu}_{\mathcal{A}}(b_1), \ldots, \tilde{\mu}_{\mathcal{A}}(b_n), \tilde{s}_2\} \geq rmax\{\tilde{s}, \tilde{s}_2\} \geq \tilde{s} > \tilde{s}_1$. Then we get $rmax\{\tilde{\mu}_{\mathcal{A}}(g(b_1^n)), \tilde{s}_1\} \geq \tilde{s}$. Therefore $\tilde{\mu}_{\mathcal{A}}(g(b_1^n)) \geq \tilde{s}$ which means $g(b_1^n) \in F(\mathcal{A}; \tilde{s})$. Thus $F(\mathcal{A}; \tilde{s})$ is a hyperideal of $\mathcal{R}$ for every $\tilde{s}_1 < \tilde{s} \leq \tilde{s}_2$.

$\Longleftarrow$ Let $rmax\{\tilde{\mu}_{\mathcal{A}}(a), \tilde{s}_1\} < rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m), \tilde{s}_2\} = \tilde{s}$ for some $a \in f(a_1^m)$ such that $a_1^m \in \mathcal{R}$. This means $\tilde{s}_1 < \tilde{s} \leq \tilde{s}_2$ and $a_1^m \in F(\mathcal{A}; \tilde{s})$. Then $f(a_1^m) \subseteq F(\mathcal{A}; \tilde{s})$, as $F(\mathcal{A}; \tilde{s})$ is a hyperideal of $\mathcal{R}$. Therefore we get $\tilde{\mu}_{\mathcal{A}}(a) \geq \tilde{s}$ for all $a \in f(a_1^m)$, a contradiction. Then for all $a_1^m \in \mathcal{R}$, $rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m), \tilde{s}_2\} \leq rmax\{\tilde{\mu}_{\mathcal{A}}(a), \tilde{s}_1\}$. Also, put $\tilde{s} = rmax\{\tilde{\mu}_{\mathcal{A}}(-b), \tilde{s}_1\}$ for some $b \in \mathcal{R}$. Suppose that $rmin\{\tilde{\mu}_{\mathcal{A}}(b), \tilde{s}_2\} > \tilde{s}$. Hence $b \in F(\mathcal{A}; \tilde{s})$ which implies $-b \in F(\mathcal{A}; \tilde{s})$ and so $\tilde{\mu}_{\mathcal{A}}(-b) \geq \tilde{s}$, a contradiction. Thus $rmin\{\tilde{\mu}_{\mathcal{A}}(b), \tilde{s}_2\} \leq rmax\{\tilde{\mu}_{\mathcal{A}}(-b), \tilde{s}_1\}$ for $b \in \mathcal{R}$. Now, put $\tilde{s} = rmax\{\tilde{\mu}_{\mathcal{A}}(b_1), \ldots, \tilde{\mu}_{\mathcal{A}}(b_n), \tilde{s}_2\}$ for some $b_1^n \in \mathcal{R}$. We assume that $rmax\{\tilde{\mu}_{\mathcal{A}}(g(b_1^n)), \tilde{s}_1\} < \tilde{s}$. Then we get $\tilde{s}_1 < \tilde{s} \leq \tilde{s}_2$ and $g(b_1^n) \in F(\mathcal{A}; \tilde{s})$. Therefore $\tilde{\mu}_{\mathcal{A}}(g(b_1^n)) \geq \tilde{s}$, a contradiction. So $rmax\{\tilde{\mu}_{\mathcal{A}}(b_1), \ldots, \tilde{\mu}_{\mathcal{A}}(b_n), \tilde{s}_2\} \leq rmax\{\tilde{\mu}_{\mathcal{A}}(g(b_1^n)), \tilde{s}_1\}$, for every $b_1^n \in \mathcal{R}$. Consequently, $\mathcal{A}$ is an n-ary interval valued fuzzy hyperideal with thresholds $(\tilde{s}_1, \tilde{s}_2)$ of $\mathcal{R}$. □

5 Implication-based interval valued fuzzy hyperideals of a Krasner (m, n)-hyperring

Logic is a study of language in arguments and persuasion. We can use it to judge the correctness of a chain of reasoning in a mathematical proof. Fuzzy logic is a generalization of set theoretic variables in terms of the linguistic variable truth. By using extension principal some operators like $\vee, \wedge, \neg, \longrightarrow$ can be applied in fuzzy logic. In the fuzzy logic, $[P]$ denotes the truth value of fuzzy proposition P. In the following, a correspondence between set-theoretical notions and fuzzy logic is shown.

$[a \in \mathcal{A}] = \mathcal{A}(a)$;
$[a \notin \mathcal{A}] = 1 - \mathcal{A}(a)$;
$[P \vee Q] = max\{[P], [Q]\}$;
$[P \wedge Q] = min\{[P], [Q]\}$;
$[\forall a\ P(a)] = inf[P(a)]$;
$[P \longrightarrow Q] = min\{1, 1 - [P] + [Q]\}$;
$\models P$ if and only if $[P] = 1$ for all valuations.

Various implication operators can been defined. In the following, we give a selection of the most important multi-valued implications, where α means the degree of membership of the premise, β the respective values for the consequence and I the resulting degree of truth for the implication.

Name	**Definition of Implication operators**
Early Zadeh	$\mathrm{I}_m(\alpha, \beta) = \max\{1 - \alpha, min\{\alpha, \beta\}\}$,
Łukasiewicz	$\mathrm{I}_a(\alpha, \beta) = \min\{1, 1 - \alpha + \beta\}$,
Kleene-Dienes	$\mathrm{I}_b(\alpha, \beta) = \max\{1 - \alpha, \beta\}$,
Contraposition of Godel	$\mathrm{I}_{cg}(\alpha, \beta) = \begin{cases} 1 & \alpha \leq \beta, \\ 1 - \alpha & \alpha > \beta, \end{cases}$
Standard Star (Godel)	$\mathrm{I}_g(\alpha, \beta) = \begin{cases} 1 & \alpha \leq \beta, \\ \beta & \alpha > \beta, \end{cases}$
Goguen	$\mathrm{I}_{gg}(\alpha, \beta) = \begin{cases} 1 & \alpha \leq \beta, \\ \frac{\beta}{\alpha} & \alpha > \beta, \end{cases}$
Gaines-Rescher	$\mathrm{I}_{gr}(\alpha, \beta) = \begin{cases} 1 & \alpha \leq \beta, \\ 0 & \alpha > \beta. \end{cases}$

In the following definition, we considered the definition of implicative operator in the Łukasiewicz system of continuous-valued logic.

Definition 5.1. *Let $\mathcal{A}$ be an interval valued fuzzy set of a Krasner (m, n)-hyperring $\mathcal{R}$. $\mathcal{A}$ refers to an n-ary interval valued fuzzifying hyperideal of $\mathcal{R}$ if it satisfies:*

(1) $\models [rmin\{[a_1 \in \mathcal{A}], \ldots, [a_m \in \mathcal{A}]\} \longrightarrow [\forall a \in f(a_1^m),\ a \in \mathcal{A}]]$, *for all* $a_1^m \in \mathcal{R}$,

(2) $\models [[b \in \mathcal{A}] \longrightarrow [-b \in \mathcal{A}]]$, *for each* $b \in \mathcal{R}$,

(3) $\models [rmax\{[b_1 \in \mathcal{A}], \ldots, [b_n \in \mathcal{A}]\} \longrightarrow [g(b_1^n) \in \mathcal{A}]]$, *for all* $b_1^n \in \mathcal{R}$.

It is clear that an interval valued fuzzifying hyperideal is an ordinary n-ary interval valued fuzzy hyperideal. We have the notion of interval valued $\tilde{t}$-tautology. In fact, $\models_{\tilde{t}} P$ if and only if $[P] \geq \tilde{t}$ for all valuations (see [31]). Now, we give the following definition.

Definition 5.2. *Let $\mathcal{A}$ be an interval valued fuzzy set of a Krasner (m, n)-hyperring $\mathcal{R}$ and $t \in (0, 1]$. Then $\mathcal{A}$ is said to be an n-ary $\tilde{t}$-implication-based interval valued fuzzy hyperideal of $\mathcal{R}$ if the following conditions hold:*

(1) $\models_{\tilde{t}} [rmin\{[a_1 \in \mathcal{A}], \ldots, [a_m \in \mathcal{A}]\} \longrightarrow [\forall a \in f(a_1^m),\ a \in \mathcal{A}]]$, *for all* $a_1^m \in \mathcal{R}$,

(2) $\models_{\tilde{t}} [[b \in \mathcal{A}] \longrightarrow [-b \in \mathcal{A}]]$, *for each* $b \in \mathcal{R}$,

(3) $\models_{\tilde{t}} [rmax\{[b_1 \in \mathcal{A}], \ldots, [b_n \in \mathcal{A}]\} \longrightarrow [g(b_1^n) \in \mathcal{A}]]$, *for all* $b_1^n \in \mathcal{R}$.

Corollary 5.3. *Let $\mathcal{A}$ be an interval valued fuzzy set of a Krasner (m, n)-hyperring $\mathcal{R}$ and I be an implicative operator. Then $\mathcal{A}$ is an n-ary $\tilde{t}$-implication-based interval valued fuzzy hyperideal of $\mathcal{R}$ for some $t \in (0, 1]$ if and only if*

(1) *for any* $a_1^m \in \mathcal{R}$, $I(rmin\{\tilde{\mu}_{\mathcal{A}}(a_1), \ldots, \tilde{\mu}_{\mathcal{A}}(a_m), rinf\{rmax\{\tilde{\mu}_{\mathcal{A}}(a), \tilde{s}_1\} \mid a \in f(a_1^m)\} \geq \tilde{t}$,

(2) *for any* $b \in \mathcal{R}$, $I(\tilde{\mu}_{\mathcal{A}}(b), \tilde{\mu}_{\mathcal{A}}(-b)) \geq \tilde{t}$,

(3) *for any* $b_1^n \in \mathcal{R}$, $I(rmax\{\tilde{\mu}_{\mathcal{A}}(b_1), \ldots, \tilde{\mu}_{\mathcal{A}}(b_n), \tilde{\mu}_{\mathcal{A}}(g(b_1^n))\} \geq \tilde{t}$.

Theorem 5.4. (1) *Suppose that $I = I_{gr}$. Then $\mathcal{A}$ is an n-ary $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $\mathcal{R}$ if and only if $\mathcal{A}$ is an n-ary interval valued fuzzy hyperideal with thresholds $(\tilde{s} = [0, 0], \tilde{r} = [1, 1])$ of $\mathcal{R}$.*

(2) *Suppose that $I = I_g$. Then $\mathcal{A}$ is an n-ary $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $\mathcal{R}$ if and only if $\mathcal{A}$ is an n-ary interval valued fuzzy hyperideal with thresholds $(\tilde{s} = [0, 0], \tilde{r} = [0.5, 0.5])$ of $\mathcal{R}$.*

(3) *Suppose that $I = I_{cg}$. Then $\mathcal{A}$ is an n-ary $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $\mathcal{R}$ if and only if $\mathcal{A}$ is an n-ary interval valued fuzzy hyperideal with thresholds $(\tilde{s} = [0.5, 0.5], \tilde{r} = [1, 1])$ of $\mathcal{R}$.*

Proof. These can be proved by using definitions. □

Corollary 5.5. (1) *Let $I = I_{gr}$. Then $\mathcal{A}$ is an n-ary $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $\mathcal{R}$ if and only if $\mathcal{A}$ is an ordinary n-ary interval valued fuzzy hyperideal of $\mathcal{R}$.*

(2) *Let $I = I_g$. Then $\mathcal{A}$ is an n-ary $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $\mathcal{R}$ if and only if $\mathcal{A}$ is an n-ary interval valued $(\in, \in \vee q)$-fuzzy hyperideal of $\mathcal{R}$.*

(3) *Let $I = I_{cg}$. Then $\mathcal{A}$ is an n-ary $[0.5, 0.5]$-implication-based interval valued fuzzy hyperideal of $\mathcal{R}$ if and only if $\mathcal{A}$ is an n-ary interval valued $(\overline{\in}, \overline{\in \vee q})$-fuzzy hyperideal of $\mathcal{R}$.*

6 Conclusion

Since hyperstructure theory was introduced by Marty in 1934, the idea has been investigated by many researches in the following decades. In this paper, we introduced and characterized the interval valued (α, β)-fuzzy hyperideals of a Krasner (m, n)-hyperring, in which special attention was concentrated on the interval valued $(\in, \in \vee q)$-fuzzy hyperideals. The consequences given in this paper can hopefully provide more realization into and a full cognition of algebraic hyperstructures and fuzzy set theory.

References

[1] R. Ameri, M. Norouzi, *Prime and primary hyperideals in Krasner (m, n)-hyperrings*, European Journal Of Combinatorics, (2013), 379–390.

[2] M. Anbarloei, *n-ary 2-absorbing and 2-absorbing primary hyperideals in Krasner (m, n)-hyperrings*, Matematicki Vesnik, 71(3) (2019), 250–262.

[3] M. Anbarloei, *Unifing the prime and primary hyperideals under one frame in a Krasner (m, n)-hyperring*, Communications in Algebra, 49 (2021), 3432–3446.

[4] A. Asadi, R. Ameri, *Direct limit of Krasner (m,n)-hyperrings*, Journal of Sciences, 31(1) (2020), 75–83.

[5] S. Corsini, *Prolegomena of hypergroup theory*, Italy: Second edition, Aviani editor, 1993.

[6] P. Corsini, V. Leoreanu, *Join spaces associated with fuzzy sets*, Journal of Combinatorics, Information System Sciences, 20(1-4) (1995), 293–303.

[7] S. Corsini, V. Leoreanu, *Applications of hyperstructure theory*, Advances in Mathematics, Kluwer Academic Publishers, Springer New York, NY, 5 (2003). DOI: 10.1007/978-1-4757-3714-1.

[8] B. Davvaz, *Fuzzy Krasner (m, n)-hyperrings*, Computers and Mathematics with Applications, 59 (2010), 3879–3891.

[9] B. Davvaz, V. Leoreanu-Fotea, *Hyperring theory and applications*, USA: International Academic Press, Palm Harbor, 2007.

[10] B. Davvaz, T. Vougiouklis, *n-ary hypergroups*, Iranian Journal of Science and Technology, 30(A2) (2006), 165–174.

[11] T.M. Dutta, S. Kar, S. Purkait, *Interval-valued fuzzy prime and semiprime ideals of a hypersemiring*, Annals of Fuzzy Mathematics and Informatics, 9(2) (2015), 261–278.

[12] M. Farshi, B. Davvaz, *Krasner $F^{(m,n)}$-hyperrings,* Iranian Journal of Fuzzy Systems, 11(6) (2014), 67–88.

[13] K. Hila, K. Naka, B. Davvaz, *On (k, n)-absorbing hyperideals in Krasner (m, n)-hyperrings,* Quarterly Journal of Mathematics, 69 (2018), 1035–1046.

[14] M. Krasner, *A class of hyperrings and hyperfields*, International Journal of Mathematics and Mathematical Sciences, 2 (1983), 307–312.

[15] V. Leoreanu-Fotea, *Canonical n-ary hypergroups*, Italian Journal of Pure and Applied Mathematics, 24(2008). https://www.researchgate.net/publication/281572515_Canonical_n-ary_hypergroups.

[16] V. Leoreanu-Fotea, B. Davvaz, *n-hypergroups and binary relations*, European Journal of Combinatorics, 29 (2008), 1027–1218.

[17] V. Leoreanu-Fotea, B. Davvaz, *Roughness in n-ary hypergroups*, Information Sciences, 178 (2008), 4114–4124.

[18] V. Leoreanu-Fotea, B. Davvaz, *Fuzzy hyperrings*, Fuzzy Sets and Systems, 160 (2009), 2366–2378.

[19] F. Marty, *Sur une generalization de la notion de groupe,* 8^{th} Congress Math. Scandenaves, Stockholm, (1934), 45–49.

[20] S. Mirvakili, B. Davvaz, *Relations on Krasner (m, n)-hyperrings*, European Journal of Combinatorics, 31 (2010), 790–802.

[21] M. Norouzi, R. Ameri, V. Leoreanu-Fotea, *Normal hyperideals in Krasner (m, n)-hyperrings*, Analele stiintifice ale Universitatii Ovidius Constanta, 26(3) (2018), 197–211.

[22] S. Ostadhadi-Dehkordi, B. Davvaz, *A note on isomorphism theorems of Krasner (m, n)- hyperrings*, Arabian Journal of Mathematics, 5 (2016), 103–115.

[23] T. Vougiouklis, *Hyperstructures and their representations*, Florida: Hadronic Press Inc., 1994.

[24] A. Yassine, M.J. Nikmehr, R. Nikandish, *n-Ary k-absorbing hyperideals in krasner (m, n)-hyperrings*, Afrika Matematika, (2022), 19–33. DOI:10.1007/s13370-022-00961-6.

[25] Y. Yin, B. Davvaz, J. Zhan, *A fuzzy view of* Γ*-hyperrings*, Neural Computing and Applications, 21(5) (2012), 979–992.

[26] L.A. Zadeh, *Fuzzy sets*, Information and Control, 8 (1965), 338–353.

[27] L.A. Zadeh, *The concept of a linguistic variable and its application to approximate reason*, Information and Control, 18 (1975), 199–249.

[28] M.M. Zahedi, A. Hasankhani, *F-polygroups I*, The Journal of Fuzzy Mathematics, 4(3) (1996), 533–548.

[29] M.M. Zahedi, A. Hasankhani, *F-polygroups II*, Information Sciences, 89(3-4) (1996), 225–243.

[30] J. Zhan, B. Davvaz, Y. Jun, *Generalized fuzzy algebraic hypersystems*, Italian Journal of Pure and Applied Mathematics, (30) (2013), 43–58.

[31] J. Zhan, B. Davvaz, K.P. Shum, *Generalized fuzzy hyperideals of hyperrings*, Computers and Mathematics with Applications, 56 (2008), 1732–1740.

[32] J. Zhan, B. Davvaz, K.P. Shum, *A new view of fuzzy hypernear-rings*, Information Sciences. An International Journal, 178(2) (2008), 425–438.