Journal of Algebraic Hyperstructures and Logical Algebras

Volume 3, Number 2, (2022), pp. 59-68

# On subhypergroups of cyclic hypergroups

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**Abstract**

The aim of this paper is to study of properties of subhypergroups of a cyclic hypergroup. It has been examined whether the theorems existing in cyclic groups exist in cyclic hypergroups. A characterization has been investigated to subhypergroups of a cyclic hypergroup be cyclic.

**Article Information**

Corresponding Author:
D. Bayrak Delice;
Received: November 2021;
Revised: December 2021;
Accepted: January 2022;
Paper type: Original.

**Keywords:**

Cyclic hypergroup, subhypergroups.

## 1 Introduction

As a generalization of algebraic structures, the hyperstructure was introduced by Marty [15] in 1934. Since then this theory has enjoyed rapid development [3, 4, 7, 8]. The cyclicity constitutes a very powerful property of a group. So, the notion of cyclicity was one of the very first topics of hypergroups. Firstly, Wall [20] worked on cyclic hypergroup. For a long time later, two different approaches to cyclicity on hypergroup were out. One of them was studied by De Salvo, Freni and Corsini [6, 9, 10, 11]. The other is studied by Vougiouklis, Konguetsof, Kessoglides and Spartalis [13, 17, 18, 19]. Finally, Novák et al. [16] study these two different cyclicity definitions were examined and revealed a comprehensive study of cyclic hypergroups. Besides these studies, Gu [12] introduced the concept of the index of a generator in cyclic hypergroups. Many researchers worked on cyclic hypergroups [2, 14]. Although many important results have been obtained, there are many open problems in cyclic hypergroups.

https://doi.org/10.52547/HATEF.JAHLA.3.2.5

This paper is organized as: We first give some fundamental definitions and results concerning hyperstructure and cyclic hypergroup that are used throughout this paper. In Section 2, we examine some open questions on subhypergroups of cyclic hypergroups.
We briefly recall some basic concepts of hypergroup and cyclic hypergroup. For more details, we refer to the references quoted from [5], [7] and [8].

## 2 Preliminaries

Now, we first introduce the lattice-theoretic base of our work. Let $L$ be a lattice where ”$\leq$” denotes the partial ordering of $L$, the join (sup) and meet (inf) of the elements of $L$ are denoted by ”$\vee$” and ”$\wedge$”, respectively. We also write 1 and 0 for top and bottom elements of $L$, respectively. We say that $L$ is a complete lattice if $L$ is closed with respect to arbitrary suprema and arbitrary infima. It is well known that a nonempty ordered set $L$ is a complete lattice if it is closed under arbitrary infima.
A filter $F$ of $L$ is a subset of $L$ such that

i) $a, b \in F \Rightarrow a \wedge b \in F$,

ii) If $a \in F$ and $b \in L$ such that $a \leq b$, then $b \in F$.

The principal filter of $L$ generated by $a$ is denoted by $F(a)$, for every $a \in L$. That is,

$$F(a) = \{x \in L \mid \ a \leq x\}.$$

Let $H$ be a nonempty set and let $\mathcal{P}^*(H)$ be the set of all nonempty subsets of $H$. Then a hyperoperation on $H$ is a map $\circ : H \times H \longrightarrow \mathcal{P}^*(H)$ and the couple $(H, \circ)$ is called a hypergroupoid. For any two nonempty subsets $A$ and $B$ of $H$ and $x \in H$, the sets $A \circ B$, $A \circ x$ and $x \circ A$ are defined by:

$$A \circ B = \bigcup \{a \circ b \mid a \in A, b \in B\},$$
$$A \circ x = A \circ \{x\},$$
$$x \circ A = \{x\} \circ A.$$

The set $A \circ B$ is called hyperproduct of $A$ and $B$.
A hypergroupoid $(H, \circ)$ is called a semihypergroup if for all $a, b, c$ of $H$ we have

$$a \circ (b \circ c) = (a \circ b) \circ c.$$

A hypergroupoid $(H, \circ)$ is called a quasihypergroup if for all $a$ of $H$ we have

$$a \circ H = H \circ a = H.$$

A hypergroup is a hypergroupoid which is both a semihypergroup and a quasihypergroup.
$(H, \circ)$ is called a commutative hypergroup of H if $x \circ y = y \circ x$ for all $x, y \in H$.
A nonempty subset K of a hypergroup $(H, \circ)$ is called a subhypergroup of H if $K \circ K \subseteq K$ and $K$ is a hypergroup under the hyperoperation $\circ$. In other words, it is a hypergroup according to the hyperoperation on $H$. $K$ provides the following conditions:

1) $a \circ b \subseteq K$ for all $a, b \in K$.

2) $a \circ K = K \circ a = K$, for all $a \in K$.

A subhypergroup $K$ of a hypergroup $(H, \circ)$ is called closed if for all $a, b \in K$ and $x \in H$, from $a \in x \circ b$ and $a \in b \circ x$, it follows that $x \in K$

Let $(H_1, \circ)$ and $(H_2, *)$ be two hypergroupoids. A map $f : H_1 \to H_2$, is called

- a homomorphism if for all $x, y \in H_1$, we have $f(x \circ y) \subseteq f(x) * f(y)$;
- a good homomorphism if for all $x, y \in H_1$, we have $f(x \circ y) = f(x) * f(y)$.

**Lemma 2.1.** [7] *Let $f : (H_1, \circ) \to (H_2, *)$ be a good homomorphism and $K$ be the subhypergroup of $H_1$. In this case, $f(K)$ is the subhypergroup of $H_2$.*

Firstly, Wall in [20] defined cyclic hypergroups. If a hypergroup $H$ is generated by a single element $a$ of $H$, then $H$ will be called a cyclic hypergroup. The definition of Vougiouklis enables to describe of as many types of cyclicity as possible.

**Definition 2.2.** [18] *A hypergroup $(H, \circ)$ is called cyclic if for some $h \in H$, there is*

$$H = h^1 \cup h^2 \cup h^3 \cup ... \cup h^n \cup ... \tag{1}$$

*where $h^1 = h$ and $h^m = \underbrace{h \circ h \circ ... \circ h}_{m}$.*

If there exists an integer $n > 0$, the minimum one with the following property

$$H = h^1 \cup h^2 \cup h^3 \cup ... \cup h^n, \tag{2}$$

then we call $H$ a cyclic hypergroup with finite period and we call $h$ a generator of $H$ with period $n$. If there is no number $n$ for which (2) is valid, but (1) is valid, then we say that $H$ has infinite period for $h$. If all generators of $H$ have the same period, then we call $H$ cyclic with period.
If there exists an integer $n > 0$, the minimum one with the following property

$$H = h^n, \tag{3}$$

then we call $H$ a single-power cyclic hypergroup and $h$ a generator of $H$ with period $n$. If (1) is valid and also for all $n \in \mathbb{N}\backslash\{0\}$ and $n \geq n_0$, for constant $n_0 \in \mathbb{N}\backslash\{0\}$, the following condition is valid

$$H = h^1 \cup h^2 \cup \cdots \cup h^{n-1} \subsetneq h^n, \tag{4}$$

then we call $H$ a single-power cyclic hypergroup with infinite period for $h$.
We define the set generated by an element $a$ is $< a >= \{a\} \cup a^2 \cup \cdots \cup a^n \cup \ldots$, for all $a \in H$.

# 3 Main results

Although the theory of hypergroups is an appropriate generalization of the classical group theory, one can see many properties do not exist in hypergroups.
One of the most basic theorems of cyclic groups is that each subgroup of them also is cyclic. However, with the following example, it is seen that the subhypergroups of the cyclic hypergroup do not have to be cyclic.

Figure 1: Hasse diagram of the lattice L

**Example 3.1.** *Let $(L, \wedge, \vee)$ be a lattice with a minimum element $0$. If for all $a \in L$, $F(a)$ denotes the principal filter generated from $a$, then we obtain a hypergroup $(L, \circ)$ $a \circ b = F(a \wedge b)$.*
*Now, let L be as following.*

*Since $L = 0^2$, $L$ is a single-power cyclic hypergroup.*
*$L\backslash\{0\}$ is a subhypergroup of $(L, \circ)$, it is not a cyclic subhypergroup. Really,*

$$a \circ b = F(a \wedge b) \subseteq L\backslash\{0\} \text{ for all } a, b \in L\backslash\{0\},$$
$$a \circ L\backslash\{0\} = \bigcup_{x \in L\backslash\{0\}} F(a \wedge x) = L\backslash\{0\}.$$

*So, $L\backslash\{0\}$ is a subhypergroup of $L$. It is not generated by any element of $L\backslash\{0\}$. Suppose that $L\backslash\{0\} =< x >$ for some $x \in L\backslash\{0\}$.*

$$< x >= \{x\} \cup x^2 \cup \cdots = F(x).$$

*Since $\frac{x}{2} < x \in F(x)$, so $\frac{x}{2} \notin F(x)$. So, $L\backslash\{0\} \neq< x >$, for all $x \in L\backslash\{0\}$. Thus $L\backslash\{0\}$ is not cyclic.*

The set of elements generated by a single element of a hypergroup is closed, but we can not say whether or not it forms a hypergroup.

**Example 3.2.** *Suppose $(Z, +, \leq)$, with the usual addition and ordering of integers. Since $(Z, +, \leq)$ is a partially ordered group, $(Z, *)$, where $a * b = \{x \in Z | a + b \leq x\}$ is a hypergroup.*
*$(Z, *)$ is a single-power cyclic hypergroup with infinite periods for infinitely many generators (yet not all because only negative integers generate it) [16].*
*For example, we can see that*

$$< 5 >= \{5\} \cup \{x \in Z | 10 \leq x\}.$$

*$10 \in< 5 >$ and $10* < 5 >= \{x \in Z | 15 \leq x\}$. Since $5 \notin 10* < 5 >$, we obtain $10* < 5 > \neq < 5 >$. Thus $< 5 >$ is not a subhypergroup.*

**Question** When is the set of elements generated by a single element of a hypergroup form a subhypergroup?
To answer this question, we must put some restrictions on hyperoperation. For this, we use the following definition.

**Definition 3.3.** [16] *A hyperoperation* $\circ$ *on* $H$ *is called extensive if for all* $a, b \in H$ *there is* $\{a, b\} \subseteq a \circ b$. *A hypergroupoid* $(H, \circ)$ *with an extensive hyperoperation is called an extensive hypergroupoid.*

**Lemma 3.4.** [16] *Every extensive semihypergroup is a hypergroup.*

**Theorem 3.5.** *Let* $(H, \circ)$ *be an extensive semihypergroup. Then the set of elements generated by a single element is a subhypergroup.*

*Proof.* The set of elements generated by a single element is a semihypergroup. Since $H$ is extensive, $< a >$ is a subhypergroup for all $a \in H$, by Lemma 3.4. □

As mentioned with the above theorem, we have not a full characterization. That is, in a hypergroup (even in a cyclic hypergroup), $H$ does not need to be an extensive hypergroup for the $< a >$ to be subhypergroup for all $a \in H$. An example of this is given below.

**Example 3.6.** *Let* $(H, \circ)$ *of order 6 as follows:*

| $\circ$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 1 | 3 | 4 | 5 | 6 |
| 3 | 3 | 3 | $\{1,2\}$ | 4 | 5 | 6 |
| 4 | 4 | 4 | 4 | 6 | $\{1,2,3\}$ | 5 |
| 5 | 5 | 5 | 5 | $\{1,2,3\}$ | 6 | 4 |
| 6 | 6 | 6 | 6 | 5 | 4 | $\{1,2,3\}$ |

*There* $< 4 >=< 5 >= H$. *So,* $(H, \circ)$ *is a cyclic hypergroup. Although* $H$ *is not an extensive hypergroup, every set of elements generated by a single element is a subhypergroup.*
$< 1 >= \{1\}$.
$< 2 >= \{1, 2\}$.
$< 3 >= \{1, 2, 3\}$.
$< 6 >= \{1, 2, 3, 6\}$.

In an extensive hypergroup, although every set of elements generated by a single element is subhypergroup, it doesn't have to be a cyclic hypergroup. This can be seen in the following example.

**Example 3.7.** *Define the following hyperoperation on the real set* $\mathbb{R}$*:*

$$x \circ y = \begin{cases} \{x\}, & if \quad x = y, \\ [\min\{x, y\}, \max\{x, y\}] & if \quad x \neq y. \end{cases}$$

*Then* $(\mathbb{R}, \circ)$ *is a hypergroup [8]. Since* $\mathbb{R}$ *is not produced by any element of its,* $(\mathbb{R}, \circ)$ *is not cyclic. Although* $(\mathbb{R}, \circ)$ *is an extensive hypergroup,* $(\mathbb{R}, \circ)$ *is not cyclic.*

Conversely to the above example, a cyclic hypergroup does not have to be an extensive hypergroup.

**Example 3.8.** *Let define the following hyperoperation on* $H = \{a, b, c, d, e\}$.

| $\oplus$ | $a$ | $b$ | $c$ | $d$ | $e$ |
|---|---|---|---|---|---|
| $a$ | $a$ | $b,c$ | $b,c$ | $d$ | $e$ |
| $b$ | $b,c$ | $d$ | $d$ | $e$ | $a$ |
| $c$ | $b,c$ | $d$ | $d$ | $e$ | $a$ |
| $d$ | $d$ | $e$ | $e$ | $a$ | $b,c$ |
| $e$ | $e$ | $a$ | $a$ | $b,c$ | $d$ |

*So, $(H,\oplus)$ is a commutative hypergroup.*

$$H =< b >=< c >=< d >=< e > .$$

*Thus $(H,\oplus)$ is a cyclic hypergroup with an infinite period, but it is not extensive.*

**Remark 3.9.** *It is known by Theorem 3.5 that if $H$ is an extensive hypergroup, then the set $< a >$, for all $a \in H$ is subhypergroup. But, the converse may not be true, in general. Example 3.1 is an example of this. Although $L\backslash\{0\}$ is a subhypergroup of the extensive cyclic hypergroup $(L,\circ)$, it is not cyclic.*

**Remark 3.10.** *It is known from Lagrange's Theorem that, in a finite group, the order of each subgroup divides the order of the group. In finite hypergroups, is this theorem valid? Example 3.6 shows that the answer to this question is negative. Clearly, in Example 3.6, the order of the hypergroup $H$ is 6, while the order of the subhypergroup $< 6 >$ is 4.*

While every group of prime order is cyclic, a hypergroup have prime order may not be cyclic. An example of this can be seen in the following.

**Example 3.11.** *The hyperoperation $\circ$ on $H = \{1,2,3,4,5,6,7\}$ is defined as:*

| $\circ$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | {6,7} | {6,7} |
| 2 | 2 | 1 | 5 | {6,7} | 3 | 4 | 4 |
| 3 | 3 | {6,7} | 1 | 5 | 4 | 2 | 2 |
| 4 | 4 | 5 | {6,7} | 1 | 2 | 3 | 3 |
| 5 | 5 | 4 | 2 | 3 | {6,7} | 1 | 1 |
| 6 | {6,7} | 3 | 4 | 2 | 1 | 5 | 5 |
| 7 | {6,7} | 3 | 4 | 2 | 1 | 5 | 5 |

*$(H,\circ)$ is hypergroup [1]. We can obtain,*

$$< 1 >=\{1\},$$

$$< 2 >=\{1,2\},$$

$$< 3 >=\{1,3\},$$

$$< 4 >=\{1,4\},$$

$$< 5 >=< 6 >=< 7 >=\{1,5,6,7\},$$

*Thus $H$ isn't produced by any elements of its. As a result, although $H$ has prime order, it is not a cyclic hypergroup.*

Let $G$ be a group, $\emptyset \neq K \subseteq G$ . The intersection of all subgroups of $G$ containing $K$ is called the subgroup produced by $K$ and denoted by $< K >$.

$$< K >= \bigcap_{\substack{K \subseteq L \\ L \leq G}} L.$$

As seen from the definition, the $< K >$ subgroup is the smallest subgroup that contains $K$. If $K = \{a\}$ is singleton, $< K > = < a > = \{a^n | \; n \in \mathbb{Z}\}$ and this definition is the same as the definition of subgroup generated by an element.
In a group, the sets $< a > = \{a^n | \; n \in \mathbb{Z}\}$ and $< a >= \bigcap \{L \mid a \in L, \;\; L \leq G\}$ are coincide. However, in a hypergroup $H$, the set $< a > = \{a\} \cup a^2 \cup \cdots \cup a^n$ does not coincide with the set $\bigcap \{L \mid a \in L, \;\; L \in Sub(H)\}$. An example of this situation can be see in the following.

**Example 3.12.** *Defined on the partially ordered group* $(\mathbb{Z}, +, \leq )$

$$a * b = \{x \in Z | a + b \leq x\}.$$

*With the hyperoperation* $(\mathbb{Z}, *)$ *it is shown by Example 2.2 that it is a hypergroup. Clearly,*

$$< 5 >= \{5\} \cup \{x \in \mathbb{Z} | 10 \leq x\},$$

*and*

$$\bigcap_{\substack{5 \in L \\ L \leq G}} L = \{x \in \mathbb{Z} | 5 \leq x\}.$$

*Thus,*

$$< 5 > \neq \bigcap_{\substack{5 \in L \\ L \leq G}} L.$$

In a cyclic hypergroup, if the set produced by an element is a subhypergroup, it may ask whether it belongs to a special class. In this context; the following example shows that the subhypergroup produced by an element in a cyclic hypergroup does not need to be closed.

**Example 3.13.** *Let* $(A, \circ)$ *be a hypergroup, with at least two elements and let* $T = \{t_i\}_{i \in \mathbb{N}}$ *such that* $A \cap T = \emptyset$ *and* $t_i \neq t_j$*, for* $i \neq j$*. We define the hyperoperation* $\otimes$ *on* $H = A \cup T$ *as follows:*
*if* $(x, y) \in A^2$*, then* $x \otimes y = A$*;*
*if* $(x, y) \in A \times T$*, then* $x \otimes t = t \otimes x = A \backslash \{x\} \cup T$*;*
*if* $(t_i, t_j) \in T \times T$*, then* $t_i \otimes t_j = A \cup t_{\{i+j\}}$*.*
*Then* $(H, \otimes)$ *is a hypergroup [8] and all elements of* $T$ *are generator of* $H$*. So,* $(H, \otimes)$ *is a cyclic hypergroup with period 3.* $H = t_i \cup {t_i}^2 \cup {t_i}^3$ *for all* $t_i \in T$*. Really,*
${t_i}^2 = A \cup \{t_{2i}\}$ *and*

$$\begin{aligned} {t_i}^3 &= t_i \otimes (A \cup \{t_{2i}\}) \\ &= \left(\textstyle\bigcup_{x \in \mathrm{A}} t_i \otimes x\right) \cup (t_i \otimes t_{2i}) \\ &= \left(\textstyle\bigcup_{x \in \mathrm{A}} (A \backslash \{x\}) \cup T\right) \cup (A \cup \{t_{3i}\}) \\ &= A \cup T \\ &= H. \end{aligned}$$

*So,* $H$ *is a single-power cyclic hypergroup with period 3.*
*Since* $x \otimes y = A \subseteq A$ *and* $x \otimes A = \bigcup_{a \in A} x \otimes a = A$*, for all* $x, y \in A$*,* $A$ *is a subhypergroup of* $H$*. Hence,* $\langle x \rangle = A$*, for all* $x \in A$*. Thus,* $A$ *is a cyclic subhypergroup. But, as can be seen in the following,* $A$ *is not a closed subhypergroup.*
*Let* $x, y \in A$ *and* $x \neq y$*. Then,* $x \in t \otimes y = A \backslash \{y\} \cup T$*. Since* $t \notin A$*,* $A$ *isn't a closed subhypergroup.*

The weakest of special classes of subhypergroups is a closed subhypergroup. It is clear that from the above example that subhypergroups of a cyclic hypergroup do not belong to other special classes (invertible, ultraclosed and conjugable).

Even if a cyclic hypergroup is extensive, the subhypergroup produced by an element need not be closed.

**Example 3.14.** *Example 3.1 shows that* $(L,\circ)$ *is a single-power cyclic hypergroup. Since* $\{x,\ y\} \subseteq F(\ x\wedge y) = x\circ y$ *for all* $x,\ y\in L$, $(L,\circ)$ *is an extensive cyclic hypergroup. Let the order between them be* $x < \ a < \ b$ *for* $x,\ a,\ b\in L$ *. In this case, the* $< a >$ *subhypergroup is not closed. Since* $< a > = F(a)$, $a$ *and* $b$ *are elements of the* $< a >$ *subhypergroup. Although* $a\in x\circ b$, $x \notin < a >$. *Thus* $< a >$ *is not closed.*

While many theorems in cyclic groups are not provided in hypergroups, we obtain parallel results with group theory for homomorphic images of hypergroups.

**Lemma 3.15.** *Let* $(H,\circ)$ *and* $(K,*)$ *hypergroups and* $f:\ H\to K$ *be a good homomorphism. In this case,* $f(< a >) =< f(a) >$ *for each* $a\in H$.

*Proof.* Let $x\in f(< a >)$. Then there is $y\in < a >$ such that $x = f(y)$. Here there is $n\in\mathbb{N}^*$ such that $y\in a^n$. Since $x\in f(a^n)$ and $f$ is a good homomorphism, $x\in f(a)^n$ is obtained. Thus $x \in < f(a) >$. It is obtained that $f(< a >)\subseteq < f(a) >$ and similarly $< f(a) > \subseteq f(< a >)$. □

The lemma above is not true for any non-good homomorphism $f$. An example of this can be seen in the following.

**Example 3.16.** *The hyperoperation defined on* $\mathbb{Z}$ *as follows:* $x\circ y = \{x+y, |x-y|\}$. *This hypergroup is denoted by* $(H(\mathbb{Z}),\circ)$. *The homomorphism*

$$f:(H(\mathbb{Z}),\circ)\to(H(\mathbb{Z}),\circ),\qquad f(n)=\begin{cases}0, & 2|n,\\ 1, & 2\nmid n\end{cases}$$

*is a non-good homomorphism* [8].
*In the hypergroup* $(H(\mathbb{Z}),\circ)$, *we obtain* $< 3 > = 3\mathbb{N}$ *and so* $f(< 3 >) = \{0,1\}$. *On the other hand*

$$< f(3) > =< 1 >= \mathbb{N}.$$

*Since* $f$ *is not a good homomorphism,* $f(< 3 >) \neq < f(3) >$.

**Theorem 3.17.** *Let* $(H,\circ)$ *and* $(K,*)$ *be two hypergroups and* $f:\ H\to K$ *be a good homomorphism. In this case, the good homomorphic image of the cyclic subhypergroup of* $H$ *is the cyclic subhypergroup of* $K$.

*Proof.* Let $< a >$ be a subhypergroup for $a\in H$. Since $f$ is a good homomorphism, $f(< a >)$ is a subhypergroup, by Lemma 2.1. From Lemma 3.15, $f(< a >) =< f(a) >$ so that $f(< a >)$ is a cyclic subhypergroup. □

**Theorem 3.18.** *Let*$(H,\circ)$ *and* $(K,*)$ *be hypergroups and* $H =< h >$ *be a cyclic hypergroup. If* $f:\ H\to K$ *is an epimorphism, then* $K$ *is a cyclic hypergroup.*

*Proof.* Since $f(h)\in K$, we get $< f(h) > \subseteq K$. There exists $a\in H$ such that $f(a) = k$ for all $k\in K$. Thus $a \in < h >$, and so $a\in h^l$ for some $n\in\mathbb{N}$. Hence,

$$k = f(a)\in f(h^l)\subseteq f(h)^l \subseteq < f(h) >,$$

and so $K \subseteq < f(h) >$ is obtained. Thus $K =< f(h) >$. Therefore, $K$ is a cyclic hypergroup. □

## 4 Conclusions

Although the theory of hypergroups is an appropriate generalization of the classical group theory, many properties do not exist in hypergroups. With many examples, it has been found that most of the theorems of group theory are not true for hypergroups. Some of these as follows.
As shown in our previous examples, it has been observed that the subhypergroups of the cyclic hypergroup need not be cyclic. The question "When will the set produced by a single element of a hypergroup become a subhypergroup?" has been investigated. It has been found to be true for a special class of hypergroups (extensive hypergroups). However, this does not give us a characterization. That is, in a hypergroup (even in a cyclic hypergroup), $H$ does not need to be an extensive hypergroup for the set $< a >$ to be subhypergroup for all $a \in H$.
It has been shown that Lagrange's Theorem does not work in the theory of hypergroups. While every group of prime order is cyclic, a hypergroup that has prime order may not be cyclic.
It has been shown that a cyclic hypergroup does not belong to special classes (like closed, invertible, utraclosed, conjugable) of subhypergroups.
These will surely be the subject of some further research.

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