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A note on UP-hyperalgebras

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Abstract

We introduce the concept of UP-hyperalgebras which is a generalization of UP-algebras, and investigate some related properties. Moreover, we introduce the concepts of UP-hypersubalgebras, UP-hyperideals of types 1 and 2, and s-UP-hyperideals of types 1 and 2 in UP-hyperalgebras and give some relations among these concepts. We try to show that these concepts are independent by some examples. Furthermore, the closed condition and the R-condition of a nonempty subset are discussed.

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1 Introduction

A type of the logical algebra, a UP-algebra was introduced by Iampan [8], and it is known that the class of KU-algebras is a proper subclass of the class of UP-algebras. It has been studied and examined by many researchers, for example, Romano [15, 16, 18] studied UP-ideals, proper UP-filters, and some their decompositions in UP-algebras. Senapati et al. [20, 21] studies applied cubic set and interval-valued intuitionistic fuzzy structure in UP-algebras. Ansari et al. [1, 2] introduced the concept of graphs associated with commutative UP-algebras and the concept of roughness in UP-algebras. Gomisong and Isla [7] established some structural properties of f-UP-semigroups. Satirad and Iampan [19] introduced the concept of topological UP-algebras and several types of subsets of topological UP-algebras. Hyperstructures have many applications to several sectors of both pure and applied sciences. The concept of hyperstructures (called also

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multialgebras) was introduced by Marty [12] in 1934. Now, the theory of algebraic hyperstructures had become a well-established branch in algebraic theory, and had been widely applied in many branches of mathematics and applied sciences [5, 6, 26, 27, 28]. In 2000, Jun et al. [11] introduced the concepts of hyper BCK-algebras, hyper BCK-ideals and weak hyper BCK-ideals, and studied the relationship between hyper BCK-ideals and weak hyper BCK-ideals. Borzooei et al. [4] introduced hyper K-algebras. In 2001, 2006, Zahedi et al. [23, 25] introduced and studied (weak) hyper K-ideals, commutative hyper K-ideals and defined simple hyper K-algebras of order 3 and quasi-commutative hyper K-algebras. In 2006, Jun et al. [10] studied hyper BCC-algebras, and introduced the concept of hyper BCC-ideals and also analyzed the relationship between hyper BCC-ideals and hyper BCK-ideals. Borzooei et al. [3] introduced the concepts of hyper BCC-algebras and hyper BCC-ideals, and studied their relationship, and then they pointed out the open problem about the relationship between hyper BCC-ideals of type 2 and weak hyper BCK-ideals. Xin [24] introduced the concept of a hyper BCI-algebra which is a generalization of a BCI-algebra, and investigated some related properties. Moreover, he introduced a hyper BCI-ideal, weak hyper BCI-ideal, strong hyper BCI-ideal and reflexive hyper BCI-ideal in hyper BCI-algebras, and gave some relations among these hyper BCI-ideals. In 2014, Radfar et al. [14] introduced the concept of hyper BE-algebras and defined some types of hyper-filters in hyper BE-algebras. In 2017, Mostafa et al. [13] introduced the concept of hyper KU-algebras and some types of hyper KU-algebras are studied. Also, a homomorphism of hyper KU-algebras is obtained. In 2019, Romano [17] introduced the concept of hyper UP-algebras and UP-hyperideals.

The goal of this paper is to generalize the concept of UP-algebras by considering the concept of binary hyperoperations, define UP-hypersubalgebras, UP-hyperideals of types 1 and 2, and s-UP-hyperideals of types 1 and 2 in this structure and describe the relationship between them. We try to show that these concepts are independent by some examples. Furthermore, the closed condition and the R-condition of a nonempty subset are discussed.

2 Preliminaries

Before we begin our study, we will give the definition and useful properties of UP-algebras.

Definition 2.1. [8] *An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a UP-algebra, where A is a nonempty set, $\cdot$ is a binary operation on A, and 0 is a fixed element of A (i.e., a nullary operation) if it satisfies the following axioms:*

(UP-1) $(\forall x, y, z \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,

(UP-2) $(\forall x \in A)(0 \cdot x = x)$,

(UP-3) $(\forall x \in A)(x \cdot 0 = 0)$, *and*

(UP-4) $(\forall x, y \in A)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y)$.

In a UP-algebra $A = (A, \cdot, 0)$, the following assertions are valid (see [8, 9]).

$$(\forall x \in A)(x \cdot x = 0), \tag{2.1}$$

$$(\forall x, y, z \in A)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0), \tag{2.2}$$

$$(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0), \tag{2.3}$$

$$(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0), \tag{2.4}$$

$$(\forall x, y \in A)(x \cdot (y \cdot x) = 0), \tag{2.5}$$

(2.6)

$$(\forall x, y \in A)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x), \tag{2.7}$$
$$(\forall x, y \in A)(x \cdot (y \cdot y) = 0), \tag{2.8}$$
$$(\forall a, x, y, z \in A)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0), \tag{2.9}$$
$$(\forall a, x, y, z \in A)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0), \tag{2.10}$$
$$(\forall x, y, z \in A)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0), \tag{2.11}$$
$$(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0), \tag{2.12}$$
$$(\forall x, y, z \in A)(((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0), \text{ and} \tag{2.13}$$
$$(\forall a, x, y, z \in A)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0). \tag{2.14}$$

From [8], the binary relation $\leq$ on a UP-algebra $A = (A, \cdot, 0)$ is defined as follows:

$$(\forall x, y \in A)(x \leq y \Leftrightarrow x \cdot y = 0). \tag{2.15}$$

In UP-algebras, 2 types of special subsets are defined as follows.

Definition 2.2. [8] *A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is called*

(1) *a UP-subalgebra of A if $(\forall x, y \in S)(x \cdot y \in S)$.*

(2) *a UP-ideal of A if*

   (i) *the constant 0 of A is in S, and*

   (ii) $(\forall x, y, z \in A)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$.

Iampan [8] proved that the concept of UP-subalgebras is a generalization of UP-ideals.

3 UP-hyperalgebras and UP-hypersubalgebras

In this section, we introduce the concepts of UP-hyperalgebras and their UP-hypersubalgebras, and investigate some properties.

Definition 3.1. [5] *Let H be a nonempty set and $\mathcal{P}^*(H)$ be the family of all nonempty subsets of H. Functions $\circ_{i_H} : H \times H \to \mathcal{P}^*(H)$, where $i \in \{1, 2, \ldots, n\}$ and n a positive number are called binary hyperoperations on H. For all $x, y \in H, \circ_{i_H}(x, y)$ is called the hyperproduct of x and y. An algebraic system $(H, \circ_{1_H}, \circ_{2_H}, \ldots, \circ_{n_H})$ is called an n-algebraic hyperstructure and structure $(H, \circ_H)$ endowed with only one binary hyperoperation is called a hypergroupoid. For any two nonempty subsets A and B of hypergroupoid H and $x \in H$, we define their hyperproduct by*

$$A \circ_H B = \bigcup_{a \in A, b \in B} a \circ_H b, A \circ_H x = A \circ_H \{x\} \text{ and } x \circ_H B = \{x\} \circ_H B.$$

Definition 3.2. *A hyperstructure $H = (H, \circ, 0)$ is called a UP-hyperalgebra, where H is a nonempty set, $\circ$ is a binary hyperoperation on H, and 0 is a fixed element of H (i.e., a nullary operation) if it satisfies the following axioms:*

(UPh-1) $(\forall x, y, z \in H)(y \circ z \ll (x \circ y) \circ (x \circ z))$,

(UPh-2) $(\forall x \in H)(x \in 0 \circ x)$,

(UPh-3) $(\forall x \in H)(x \ll 0)$, *and*

(UPh-4) $(\forall x, y \in H)(x \ll y, y \ll x \Rightarrow x = y)$,

where $x \ll y$ *is defined by* $0 \in x \circ y$ *for all* $x, y \in H$ *and for every* $A, B \subseteq H, A \ll B$ *is defined by for each* $a \in A$*, there exists* $b \in B$ *such that* $a \ll b$*. We shall use* $A \ll x$ *and* $x \ll A$ *instead of* $A \ll \{x\}$*, or* $\{x\} \ll A$*, respectively.*

Example 3.3. *Let* $H = \{0, 1, 2\}$ *be a set with a binary hyperoperation* $\circ$ *defined by the following Cayley tables:*

$\circ$	0	1	2
0	$\{0,2\}$	$\{1\}$	$\{1,2\}$
1	$\{0,1\}$	$\{1,2\}$	$\{0,1\}$
2	$\{0,2\}$	$\{2\}$	$\{0,1,2\}$

Then $(H, \circ, 0)$ *is a UP-hyperalgebra.*

Theorem 3.4. *Let* X *be a nonempty totally ordered set containing the minimum element* 0*. Define a binary hyperoperation* $\circ_X$ *on* X *by*

$$(\forall x, y \in X)\left(x \circ_X y = \begin{cases} \{0, y\} & \text{if } x \geq y, \\ \{y\} & \text{otherwise} \end{cases}\right).$$

Then $(X, \circ_X, 0)$ *is a UP-hyperalgebra.*

Proof. **UPh-2:** For all $x \in X, 0 \circ_X x$ is $\{0, x\}$ or $\{x\}$ and so $x \in 0 \circ_X x$.

UPh-3: For all $x \in X, x \circ_X 0 = \{0\}$ and so $x \ll 0$.

UPh-1: Let $x, y, z \in X$. If $0 \in (x \circ_X y) \circ_X (x \circ_X z)$, then it follows from 3.2 that $y \circ_X z \ll (x \circ_X y) \circ_X (x \circ_X z)$. If $0 \notin (x \circ_X y) \circ_X (x \circ_X z)$, then $0 \notin x \circ_X z$. Thus $x \circ_X z = \{z\}$ where $z \neq 0$, so $(x \circ_X y) \circ_X (x \circ_X z) = \{z\}$. By the definition of $\circ_X$, we have $y \in x \circ_X y$ and $z \in x \circ_X z$. Thus $y \circ_X z \subseteq (x \circ_X y) \circ_X (x \circ_X z) = \{z\}$, so $y \circ_X z = \{z\}$. Since $z \circ_X z = \{0, z\}$, we have $z \ll z$ and so $y \circ_X z \ll (x \circ_X y) \circ_X (x \circ_X z)$.

UPh-4: Let $x, y \in X$ be such that $x \neq y$. Then we may assume that $x < y$. Then $x \circ_X y = \{y\}$. Since 0 is the minimum element of X, we have $y \neq 0$. Thus $0 \notin x \circ_X y$, that is, $x \not\ll y$.

Therefore, $(X, \circ_X, 0)$ is a UP-hyperalgebra. □

Example 3.5. *By Theorem 3.4, we have* $(\mathbb{N}_0, \circ_{\mathbb{N}_0}, 0)$ *is a UP-hyperalgebra.*

Theorems 3.6, 3.8, and 3.10 can be prove in the similar way as Theorem 3.4.

Theorem 3.6. *Let* X *be a nonempty totally ordered set containing the minimum element* 0*. Define a binary hyperoperation* $\diamond_X$ *on* X *by*

$$(\forall x, y \in X)\left(x \diamond_X y = \begin{cases} X & \text{if } x \geq y, \\ \{y\} & \text{otherwise} \end{cases}\right).$$

Then $(X, \diamond_X, 0)$ *is a UP-hyperalgebra.*

Example 3.7. *By Theorem 3.6, we have* $(\mathbb{R}_{\geq 0}, \diamond_{\mathbb{R}_{\geq 0}}, 0)$ *is a UP-hyperalgebra, where* $\mathbb{R}_{\geq 0}$ *is the set of all nonnegative real numbers.*

Theorem 3.8. *Let* X *be a nonempty totally ordered set containing the maximum element* 1*. Define a binary hyperoperation* $\circ^X$ *on* X *by*

$$(\forall x, y \in X)\left(x \circ^X y = \begin{cases} \{1, y\} & \text{if } x \leq y, \\ \{y\} & \text{otherwise} \end{cases}\right).$$

Then $(X, \circ^X, 1)$ *is a UP-hyperalgebra.*

Example 3.9. *By Theorem 3.8, we have* $(\mathbb{Z}_{\leq 0}, \circ^{\mathbb{Z}_{\leq 0}}, 0)$ *is a UP-hyperalgebra, where* $\mathbb{Z}_{\leq 0}$ *is the set of all negative integers with zero.*

Theorem 3.10. *Let* X *be a nonempty totally ordered set containing the maximum element* 1*. Define a binary hyperoperation* $\diamond^X$ *on* X *by*

$$(\forall x, y \in X)\left(x \diamond^X y = \begin{cases} X & \text{if } x \leq y, \\ \{y\} & \text{otherwise} \end{cases}\right).$$

Then $(X, \diamond^X, 1)$ *is a UP-hyperalgebra.*

Example 3.11. *By Theorem 3.10, we have* $(\mathbb{R}_{\leq 0}, \diamond^{\mathbb{R}_{\leq 0}}, 0)$ *is a UP-hyperalgebra, where* $\mathbb{R}_{\leq 0}$ *is the set of all negative real numbers with zero.*

Using the axioms of a UP-algebra, we have the following theorem.

Theorem 3.12. *Let* $H = (H, \cdot, 0)$ *be a UP-algebra. Define a binary hyperoperation* $\circ$ *on* H *by*

$$(\forall x, y \in H)\,(x \circ y = \{x \cdot y\})\,.$$

Then $(H, \circ, 0)$ *is a UP-hyperalgebra.*

By Theorem 3.12, we have the following corollary.

Corollary 3.13. *Every UP-algebra induces a UP-hyperalgebra.*

From now on, unless another thing is stated, we take $H = (H, \circ, 0)$ as a UP-hyperalgebra.

Proposition 3.14. *In a UP-hyperalgebra* H*, the following properties hold: for all* $x, y, z \in H$ *and for all nonempty subsets* $A, B, C,$ *and* D *of* H*,*

(1) $0 \ll 0$,

(2) $y \ll (x \circ 0) \circ (x \circ y)$,

(3) $0 \ll (x \circ y) \circ (x \circ 0)$,

(4) $x \ll y, x \neq y \Rightarrow y \not\ll x$,

(5) $x \ll y \Rightarrow z \circ x \ll z \circ y$,

(6) $0 \circ x \ll A \Rightarrow x \ll A$,

(7) $A \subseteq 0 \circ A$,

(8) $A \ll 0$,

(9) $0 \ll A \Rightarrow 0 \in A$,

(10) $0 \in B \Rightarrow A \ll B$,

(11) $A \subseteq B, C \subseteq D \Rightarrow A \circ C \subseteq B \circ D$,

(12) $0 \in A \Rightarrow B \subseteq A \circ B$,

(13) $A \subseteq B \ll C \Rightarrow A \ll C$, *and*

(14) $(\forall a \in A, \forall b \in B)(A \circ B \ll C \Rightarrow a \circ b \ll C)$.

Proof. (1) It is straightforward by 3.2.

(2) By 3.2, we have $0 \circ y \ll (x \circ 0) \circ (x \circ y)$. By 3.2, we have $y \in 0 \circ y$. Thus $y \ll (x \circ 0) \circ (x \circ y)$.

(3) By 3.2, we have $y \circ 0 \ll (x \circ y) \circ (x \circ 0)$. By 3.2, we have $0 \in y \circ 0$. Thus $0 \ll (x \circ y) \circ (x \circ 0)$.

(4) It is straightforward by 3.2.

(5) Assume that $x \ll y$. By 3.2, we have $0 \in x \circ y \ll (z \circ x) \circ (z \circ y)$. Then $0 \ll a$ for some $a \in (z \circ x) \circ (z \circ y)$. By 3.2 and 3.2, we have $a = 0$. Thus $z \circ x \ll z \circ y$.

(6) It is straightforward by 3.2 and the definition of $\ll$.

(7) By 3.2, we have $a \in 0 \circ a \subseteq \bigcup_{a \in A} 0 \circ a = 0 \circ A$ for all $a \in A$. Thus $A \subseteq 0 \circ A$.

(8) By 3.2, we have $A \ll 0$.

(9) Assume that $0 \ll A$. Then $0 \ll a$ for some $a \in A$. By 3.2 and 3.2, we have $a = 0$ and so $0 \in A$.

(10) Assume that $0 \in B$. Then, by 3.2, we have $A \ll B$.

(11) Assume that $A \subseteq B$ and $C \subseteq D$. Then $A \circ C = \bigcup_{\substack{a \in A \subseteq B \\ c \in C \subseteq D}} a \circ c \subseteq \bigcup_{\substack{b \in B \\ d \in D}} b \circ d = B \circ D$.

(12) It follows from (7) and (11).

(13) It is straightforward by the definition of $\ll$.

(14) It follows from (13). □

Definition 3.15. *A subset S of H is called a UP-hypersubalgebra of H if the constant 0 of H is in S, and $(S, \circ, 0)$ itself forms a UP-hyperalgebra. Clearly, H is a UP-hypersubalgebra of H.*

The following example shows that the singleton $\{0\}$ is not a UP-hypersubalgebra of a UP-hyperalgebra in general.

Example 3.16. *From Example 3.3, we have $(H, \circ, 0)$ is a UP-hyperalgebra. Since $0 \circ 0 = \{0, 2\} \notin \mathcal{P}^*(\{0\})$, we have $\circ$ is not a binary hyperoperation on $\{0\}$. Hence, $(\{0\}, \circ, 0)$ is not a UP-hypersubalgebra of H.*

Example 3.17. *From Example 3.7, we have $(\mathbb{R}_{\geq 0}, \diamond_{\mathbb{R}_{\geq 0}}, 0)$ is a UP-hyperalgebra. Since $1 \diamond_{\mathbb{R}_{\geq 0}} 1 = \mathbb{R}_{\geq 0} \notin \mathcal{P}^*(\mathbb{N}_0)$, we have $\diamond_{\mathbb{R}_{\geq 0}}$ is not a binary hyperoperation on $\mathbb{N}_0$. Hence, $(\mathbb{N}_0, \diamond_{\mathbb{R}_{\geq 0}}, 0)$ is not a UP-hypersubalgebra of $\mathbb{R}_{\geq 0}$. But by Theorem 3.6, we have $(\mathbb{N}_0, \diamond_{\mathbb{N}_0}, 0)$ is a UP-hyperalgebra.*

Proposition 3.18. *Let S be a nonempty subset of H. If $x \ll y$ and $x \circ y \subseteq S$ for some $x, y \in S$, then $0 \in S$.*

Proof. Assume that $x \ll y$ and $x \circ y \subseteq S$ for some $x, y \in S$. Then $0 \in x \circ y \subseteq S$. □

Theorem 3.19. *Let S be a nonempty subset of H. Then the following statements hold:*

(1) if S is a UP-hypersubalgebra of H, then $S \circ S = S$,

(2) if S is a UP-hypersubalgebra of H, then $S \circ S$ is also a UP-hypersubalgebra of H,

(3) if $S \circ S \subseteq S$ and $0 \in S$, then S is a UP-hypersubalgebra of H, and

(4) if $S \circ S \subseteq S$ and $x \ll y$ for some $x, y \in S$, then S is a UP-hypersubalgebra of H.

Proof. (1) It is straightforward by the binary hyperoperation on S and 3.2.

(2) It follows from (1).

(3) Obviously from the definition of UP-hyperalgebras.

(4) Assume that $S \circ S \subseteq S$ and $x \ll y$ for some $x, y \in S$. By Proposition 3.18, we have $0 \in S$. It follows from (3) that S is a UP-hypersubalgebra of H. □

Theorem 3.20. *Let $\mathscr{S}$ be a nonempty family of UP-hypersubalgebras of H. Then $\bigcap_{S\in\mathscr{S}} S$ is a UP-hypersubalgebra of H.*

Proof. Clearly, $0 \in S$ for all $S \in \mathscr{S}$. Then $0 \in \bigcap_{S\in\mathscr{S}} S$. Let $x, y \in \bigcap_{S\in\mathscr{S}} S$. Then $x, y \in S$ for all $S \in \mathscr{S}$. Since S is a UP-hypersubalgebra of H, it follows from Theorem 3.19 (1) that $x \circ y \subseteq S$ for all $S \in \mathscr{S}$ and so $x \circ y \subseteq \bigcap_{S\in\mathscr{S}} S$. By Theorem 3.19 (3), we have $\bigcap_{S\in\mathscr{S}} S$ is a UP-hypersubalgebra of H. □

Remark 3.21. *The union of two UP-hypersubalgebras of a UP-hyperalgebra need not be a UP-hypersubalgebra. We show the remark with Example 3.22.*

Example 3.22. *Let $H = \{0, 1, 2, 3\}$ be a set with a binary hyperoperation $\circ$ defined by the following Cayley table:*

$\circ$	0	1	2	3
0	$\{0\}$	$\{1\}$	$\{2\}$	$\{2,3\}$
1	$\{0,1\}$	$\{0,1\}$	$\{2,3\}$	$\{1,2,3\}$
2	$\{0,2\}$	$\{0,1,3\}$	$\{0,2\}$	$\{1,2,3\}$
3	$\{0,3\}$	$\{1,2\}$	$\{0,1,3\}$	$\{2,3\}$

Then $(H, \circ, 0)$ is a UP-hyperalgebra and $\{0, 1\}$ and $\{0, 2\}$ are UP-hypersubalgebras of H. Since $1, 2 \in \{0, 1, 2\} = \{0, 1\} \cup \{0, 2\}$ but $2 \circ 1 = \{0, 1, 3\} \notin \mathcal{P}^*(\{0, 1, 2\})$, we have $\circ$ is not a binary hyperoperation on $\{0, 1, 2\}$. Hence, $(\{0, 1, 2\}, \circ, 0)$ is not a UP-hypersubalgebra of H.

Remark 3.23. *Every UP-subalgebra of a UP-algebra H is a UP-hypersubalgebra of the UP-hyperalgebra H, which is defined in Theorem 3.12.*

Proof. Let S be a UP-subalgebra of a UP-algebra $H = (H, \cdot, 0)$. Then $0 \in S$. Let $x, y \in S$. Since S is a UP-subalgebra of H, we have $x \cdot y \in S$. Thus $x \circ y = \{x \cdot y\} \subseteq S$, so $S \circ S \subseteq S$. By Theorem 3.19 (3), we have S is a UP-hypersubalgebra of the UP-hyperalgebra $(H, \circ, 0)$ in Theorem 3.12. □

Remark 3.24. *The hyperproduct of two UP-hypersubalgebras of a UP-hyperalgebra need not be a UP-hypersubalgebra. We show the remark with Example 3.25.*

Example 3.25. *Let $H = \{0, 1, 2, 3, 4, 5, 6\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	0	0	2	3	2	3	6
2	0	1	0	3	1	5	3
3	0	1	2	0	4	1	2
4	0	0	0	3	0	3	3
5	0	0	2	0	2	0	2
6	0	1	0	0	1	1	0

Then $\{0, 2\}$ and $\{0, 4\}$ are UP-subalgebras of H. By Remark 3.23, we have $\{0, 2\}$ and $\{0, 4\}$ are UP-hypersubalgebras of the UP-hyperalgebra H, which is defined in Theorem 3.12. Since $1, 4 \in \{0, 1, 4\} = \{0, 2\} \circ \{0, 4\}$ but $1 \circ 4 = \{2\} \notin \mathcal{P}^*(\{0, 1, 4\})$, we have $\circ$ is not a binary hyperoperation on $\{0, 1, 4\}$. Hence, $(\{0, 1, 4\}, \circ, 0)$ is not a UP-hypersubalgebra of H.

For the study of hyper BCC-algebras, hyper BCI-algebras, and hyper BCK-algebras, some subsets have been defined. The results of the study can be summarized as follows.

In a hyper BCC-algebra $(H, \circ, 0)$ [3], the set $S(H) := \{x \in H \mid x \circ x = \{0\}\}$ is a hyper BCC-algebra.

In a hyper BCI-algebra $(H, \circ, 0)$ [24], the set $S(H) := \{x \in H \mid 0 \circ x = \{0\}\}$ is a hyper BCI-algebra if $S(H)$ is nonempty, the set $S_K := \{x \in H \mid x \circ (x \circ 0) = \{0\}\}$ is a hyper BCI-algebra and also a hyper BCK-algebra if S_K is nonempty, and the set $S_I := \{x \in H \mid x \circ x = \{0\}\}$ is a hyper BCI-algebra if S_I is nonempty.

For a UP-hyperalgebra $(H, \circ, 0)$, we define the subsets as follows in the previous study:

$$\begin{aligned}
S_H &= \{x \in H \mid x \ll x\}, \\
S_Z &= \{x \in H \mid x \circ x = \{0\}\}, \\
S_{LI} &= \{x \in H \mid 0 \circ x = \{x\}\}, \\
S_{RZ} &= \{x \in H \mid x \circ 0 = \{0\}\}, \\
S_K &= \{x \in H \mid x \circ (x \circ 0) = \{0\}\}.
\end{aligned}$$

The following example shows that there is an S_H for some UP-hyperalgebras which is neither an empty subset nor a UP-hypersubalgebra.

Example 3.26. *From Example 3.3, we have $(H, \circ, 0)$ is a UP-hyperalgebra. We see that $S_H = \{0, 2\}$. Since $2 \circ 2 = \{0, 1, 2\} \notin \mathcal{P}^*(\{0, 2\}) = \mathcal{P}^*(S_H)$, we have $\circ$ is not a binary hyperoperation on S_H. Hence, $(S_H, \circ, 0)$ is not a UP-hypersubalgebra of H.*

The following example shows that there is an S_Z for some UP-hyperalgebras which is neither an empty subset nor a UP-hypersubalgebra.

Example 3.27. *Let $H = \{0, 1, 2\}$ be a set with a binary hyperoperation $\circ$ defined by the following Cayley table:*

$\circ$	0	1	2
0	$\{0, 1\}$	$\{1\}$	$\{1, 2\}$
1	$\{0, 1\}$	$\{0\}$	$\{0, 1\}$
2	$\{0, 1\}$	$\{2\}$	$\{0\}$

Then $(H, \circ, 0)$ *is a UP-hyperalgebra. We see that* $S_Z = \{1, 2\}$ *but* $0 \notin S_Z$*. Hence,* $(S_Z, \circ, 0)$ *is not a UP-hypersubalgebra of* H*.*

The following example shows that there is an S_{LI} for some UP-hyperalgebras which is neither an empty subset nor a UP-hypersubalgebra.

Example 3.28. *Let* $H = \{0, 1, 2\}$ *be a set with a binary hyperoperation* $\circ$ *defined by the following Cayley table:*

$\circ$	0	1	2
0	$\{0, 1\}$	$\{1\}$	$\{1, 2\}$
1	$\{0, 1\}$	$\{0, 1, 2\}$	$\{2\}$
2	$\{0, 1\}$	$\{0, 2\}$	$\{1, 2\}$

Then $(H, \circ, 0)$ *is a UP-hyperalgebra. We see that* $S_{LI} = \{1\}$ *but* $0 \notin S_{LI}$*. Hence,* $(S_{LI}, \circ, 0)$ *is not a UP-hypersubalgebra of* H*.*

The following example shows that there is an S_{RZ} for some UP-hyperalgebras which is neither an empty subset nor a UP-hypersubalgebra.

Example 3.29. *Let* $H = \{0, 1, 2\}$ *be a set with a binary hyperoperation* $\circ$ *defined by the following Cayley table:*

$\circ$	0	1	2
0	$\{0, 1\}$	$\{1\}$	$\{1, 2\}$
1	$\{0\}$	$\{1, 2\}$	$\{0, 1\}$
2	$\{0\}$	$\{2\}$	$\{0, 1, 2\}$

Then $(H, \circ, 0)$ *is a UP-hyperalgebra. We see that* $S_{RZ} = \{1, 2\}$ *but* $0 \notin S_{RZ}$*. Hence,* $(S_{RZ}, \circ, 0)$ *is not a UP-hypersubalgebra of* H*.*

The following example shows that there is an S_K for some UP-hyperalgebras which is neither an empty subset nor a UP-hypersubalgebra.

Example 3.30. *From Example 3.29, we see that* $S_K = \{1, 2\}$ *but* $0 \notin S_K$*. Hence,* $(S_K, \circ, 0)$ *is not a UP-hypersubalgebra of* H*.*

4 UP-hyperideals and s-UP-hyperideals

In this section, we introduce the concepts of UP-hyperideals of types 1 and 2 and s-UP-hyperideals of types 1 and 2 in UP-hyperalgebras, and give some relations among these concepts.

Definition 4.1. *A subset* I *of* H *is called*

(1) a UP-hyperideal of type 1 of H *if*

  (i) the constant 0 *of* H *is in* I*, and*

  (ii) $(\forall x, y, z \in H)(x \circ (y \circ z) \subseteq I, y \in I \Rightarrow x \circ z \subseteq I)$*.*

(2) a UP-hyperideal of type 2 of H *if*

  (i) the constant 0 *of* H *is in* I*, and*

(ii) $(\forall x, y, z \in H)(x \circ (y \circ z) \subseteq I, y \in I \Rightarrow (x \circ z) \cap I \neq \emptyset)$.

(3) a strong UP-hyperideal of type 1 of H (we shortly call an s-UP-hyperideal of type 1) if

(i) the constant 0 of H is in I, and

(ii) $(\forall x, y, z \in H)((x \circ (y \circ z)) \cap I \neq \emptyset, y \in I \Rightarrow x \circ z \subseteq I)$.

(4) a strong UP-hyperideal of type 2 of H (we shortly call an s-UP-hyperideal of type 2) if

(i) the constant 0 of H is in I, and

(ii) $(\forall x, y, z \in H)((x \circ (y \circ z)) \cap I \neq \emptyset, y \in I \Rightarrow (x \circ z) \cap I \neq \emptyset)$.

The following theorem follows directly from Definition 4.1.

Theorem 4.2. *(1) Every s-UP-hyperideal of type 1 of H is a UP-hyperideal of type 1.*

(2) Every s-UP-hyperideal of type 1 of H is an s-UP-hyperideal of type 2.

(3) Every UP-hyperideal of type 1 of H is a UP-hyperideal of type 2.

(4) Every s-UP-hyperideal of type 2 of H is a UP-hyperideal of type 2.

The following example shows that the converse of Theorem 4.2 (1) is not true in general.

Example 4.3. *Let $H = \{0, 1, 2\}$ be a set with a binary hyperoperation $\circ$ defined by the following Cayley table:*

$\circ$	0	1	2
0	$\{0\}$	$\{1\}$	$\{2\}$
1	$\{0, 2\}$	$\{0\}$	$\{0\}$
2	$\{0, 2\}$	$\{1, 2\}$	$\{0, 2\}$

Then $(H, \circ, 0)$ is a UP-hyperalgebra and $I := \{0, 2\}$ is a UP-hyperideal of type 1 of H. Since $(2 \circ (0 \circ 1)) \cap I = \{1, 2\} \cap \{0, 2\} \neq \emptyset$ and $0 \in I$, but $2 \circ 1 = \{1, 2\} \nsubseteq \{0, 2\} = I$, we have I is not an s-UP-hyperideal of type 1 of H.

The following example shows that the converse of Theorem 4.2 (2) is not true in general.

Example 4.4. *Let $H = \{0, 1, 2\}$ be a set with a binary hyperoperation $\circ$ defined by the following Cayley table:*

$\circ$	0	1	2
0	$\{0, 1\}$	$\{1\}$	$\{2\}$
1	$\{0, 1\}$	$\{0, 2\}$	$\{1, 2\}$
2	$\{0, 1\}$	$\{0, 1\}$	$\{0, 1, 2\}$

Then $(H, \circ, 0)$ is a UP-hyperalgebra and $I := \{0\}$ is an s-UP-hyperideal of type 2 of H. Since $(2 \circ (0 \circ 1)) \cap I = \{0, 1\} \cap \{0\} \neq \emptyset$ and $0 \in I$, but $2 \circ 1 = \{0, 1\} \nsubseteq \{0\} = I$, we have I is not an s-UP-hyperideal of type 1 of H.

The following example shows that the converse of Theorem 4.2 (3) is not true in general.

Example 4.5. *Let $H = \{0, 1, 2\}$ be a set with a binary hyperoperation $\circ$ defined by the following Cayley table:*

$\circ$	0	1	2
0	$\{0,1\}$	$\{1\}$	$\{1,2\}$
1	$\{0,1\}$	$\{1,2\}$	$\{0,1\}$
2	$\{0,1\}$	$\{2\}$	$\{0,1,2\}$

Then $(H, \circ, 0)$ is a UP-hyperalgebra and $I := \{0, 1\}$ is a UP-hyperideal of type 2 of H. Since $0 \circ (1 \circ 2) = \{0, 1\} \subseteq \{0, 1\} = I$ and $1 \in I$, but $0 \circ 2 = \{1, 2\} \nsubseteq \{0, 1\} = I$, we have I is not a UP-hyperideal of type 1 of H.

The following example shows that the converse of Theorem 4.2 (4) is not true in general.

Example 4.6. *From Example 4.3, it follows from Theorem 4.2 (3) that $I = \{0, 2\}$ is a UP-hyperideal of type 2 of H. Since $(0 \circ (2 \circ 1)) \cap I = \{1, 2\} \cap \{0, 2\} \neq \emptyset$ and $2 \in I$, but $(0 \circ 1) \cap I = \{1\} \cap \{0, 2\} = \emptyset$, we have I is not an s-UP-hyperideal of type 2 of H.*

By Theorem 4.2 and Examples 4.3, 4.4, 4.5, and 4.6, we have that the concept of UP-hyperideals of type 1 is a generalization of s-UP-hyperideals of type 1, s-UP-hyperideals of type 2 is a generalization of s-UP-hyperideals of type 1, hyper UP-ideals of type 2 is a generalization of hyper UP-ideals of type 1, and hyper UP-ideals of type 2 is a generalization of s-hyper UP-ideals of type 2. Then, we get the diagram of generalization of UP-hyperideals in UP-hyperalgebras as shown in Figure 1.

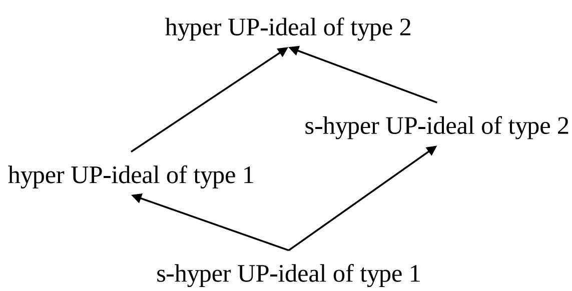

Figure 1: UP-hyperideals and s-UP-hyperideals

Example 4.7. *From Examples 4.3 and 4.6, we have $\{0, 2\}$ is a UP-hyperideal of type 1 of H but it is not an s-UP-hyperideal of type 2 of H.*

Example 4.8. *Let $H = \{0, 1, 2, 3\}$ be a set with a binary hyperoperation $\circ$ defined by the following Cayley table:*

$\circ$	0	1	2	3
0	$\{0\}$	$\{1,2\}$	$\{2,3\}$	$\{1,2,3\}$
1	$\{0,1\}$	$\{2,3\}$	$\{0,1\}$	$\{1,3\}$
2	$\{0,2,3\}$	$\{1,2\}$	$\{0,2\}$	$\{2\}$
3	H	$\{0,2,3\}$	$\{0,1,3\}$	$\{0,2\}$

Then $(H, \circ, 0)$ is a UP-hyperalgebra, and $\{0, 2, 3\}$ is an s-UP-hyperideal of type 2 of H but it is not a UP-hyperideal of type 1. Indeed, $0 \circ (2 \circ 3) = \{2, 3\} \subseteq \{0, 2, 3\}$ and $2 \in \{0, 2, 3\}$ but $0 \circ 3 = \{1, 2, 3\} \nsubseteq \{0, 2, 3\}$.

By Examples 4.7 and 4.8, we have that a UP-hyperideal of type 1 and an s-hyper UP-ideal of type 2 are not sufficient conditions for each other in general. Then, we get the diagram as shown in Figure 2.

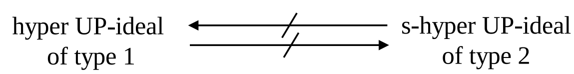

Figure 2: UP-hyperideals of type 1 and s-UP-hyperideals of type 2

Theorem 4.9. *$\{0\}$ is a UP-hyperideal of type 1 of H and also a UP-hyperideal of type 2.*

Proof. Clearly, $0 \in \{0\}$. Let $x, y, z \in H$ be such that $x \circ (y \circ z) \subseteq \{0\}$ and $y \in \{0\}$. Then $x \circ (0 \circ z) \subseteq \{0\}$. By 3.2, we have $z \in 0 \circ z$. Thus $x \circ z \subseteq x \circ (0 \circ z) \subseteq \{0\}$. Hence, $\{0\}$ is a UP-hyperideal of type 1 of H. □

The following example shows that $\{0\}$ of a UP-hyperalgebra need not be an s-UP-hyperideal of types 1 and 2.

Example 4.10. *Let $H = \{0, 1, 2, 3\}$ be a set with a binary hyperoperation $\circ$ defined by the following Cayley table:*

$\circ$	0	1	2	3
0	$\{0\}$	$\{1\}$	$\{2\}$	$\{2,3\}$
1	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{1,2,3\}$
2	$\{0,2\}$	$\{2,3\}$	$\{0,2\}$	$\{1,2,3\}$
3	$\{0,3\}$	$\{1,2\}$	$\{0,1,3\}$	$\{2,3\}$

Then $(H, \circ, 0)$ is a UP-hyperalgebra. Since $(2\circ(0\circ3))\cap\{0\} = (2\circ\{2,3\})\cap\{0\} = H\cap\{0\} = \{0\} \neq \emptyset$ and $0 \in \{0\}$ but $(2 \circ 3) \cap \{0\} = \{1, 2, 3\} \cap \{0\} = \emptyset$. Hence, $\{0\}$ is not an s-UP-hyperideal of type 2 of H and also not an s-UP-hyperideal of type 1.

Remark 4.11. *From Theorem 4.9 and Example 3.16, we have a UP-hyperideal of type 1 of H is not a UP-hypersubalgebra in general. Also, a UP-hyperideal of type 2 of H is not a UP-hypersubalgebra in general.*

Theorem 4.12. *If H is a UP-hyperalgebra satisfying the following condition:*

$$(\forall x \in H)(0 \circ x = \{x\}), \tag{4.1}$$

then $\{0\}$ is an s-UP-hyperideal of type 2 of H.

Proof. Assume that H is a UP-hyperalgebra satisfying the condition (4.1). Clearly, $0 \in \{0\}$. Let $x, y, z \in H$ be such that $(x \circ (y \circ z)) \cap \{0\} \neq \emptyset$ and $y \in \{0\}$. Then $0 \in x \circ (0 \circ z) = x \circ \{z\} = x \circ z$, so $(x \circ z) \cap \{0\} \neq \emptyset$. Hence, $\{0\}$ is an s-UP-hyperideal of type 2 of H. □

Remark 4.13. *If H is a UP-hyperalgebra with its binary hyperoperation maps to a singleton set, then $\{0\}$ is an s-UP-hyperideal of type 1 of H.*

Proof. Assume that H is a UP-hyperalgebra with its binary hyperoperation maps to a singleton set. Clearly, $0 \in \{0\}$. Let $x, y, z \in H$ be such that $(x \circ (y \circ z)) \cap \{0\} \neq \emptyset$ and $y \in \{0\}$. By assumption and 3.2, we have $\{0\} = x \circ (0 \circ z) = x \circ \{z\} = x \circ z$, that is, $x \circ z \subseteq \{0\}$. Hence, $\{0\}$ is an s-UP-hyperideal of type 1 of H. □

The following example shows that the condition: its binary hyperoperation maps to a singleton set that is necessary.

Example 4.14. *Let $H = \{0, 1, 2\}$ be a set with a binary hyperoperation $\circ$ defined by the following Cayley table:*

$\circ$	0	1	2
0	$\{0\}$	$\{1\}$	$\{2\}$
1	$\{0\}$	$\{0,1,2\}$	$\{0,2\}$
2	$\{0\}$	$\{1\}$	$\{0,2\}$

Then $(H, \circ, 0)$ is a UP-hyperalgebra but $\{0\}$ is not an s-UP-hyperideal of type 1 of H. Indeed, $(2 \circ (0 \circ 2)) \cap \{0\} = \{0, 2\} \cap \{0\} \neq \emptyset$ and $0 \in \{0\}$, but $2 \circ 2 = \{0, 2\} \nsubseteq \{0\}$.

Example 4.15. *From Example 4.14 and by Theorem 4.12, we have H is a UP-hyperalgebra satisfying the condition (4.1) and $\{0\}$ is an s-UP-hyperideal of type 2 of H but not an s-UP-hyperideal of type 1.*

Remark 4.16. *If H is a UP-hyperalgebra with its binary hyperoperation maps to a singleton set, then UP-hyperideals of type 1, UP-hyperideals of type 2, s-UP-hyperideals of type 1, and s-UP-hyperideals of type 2 of H coincide.*

Proof. Since $x \circ (y \circ z)$ is a singleton set for all $x, y, z \in H$, it is straightforward by the definition. □

Theorem 4.17. *Let $\mathscr{I}$ be a nonempty family of UP-hyperideals of type 1 of H. Then $\bigcap_{I \in \mathscr{I}} I$ is a UP-hyperideal of type 1 of H.*

Proof. Clearly, $0 \in I$ for all $I \in \mathscr{I}$. Then $0 \in \bigcap_{I \in \mathscr{I}} I$. Let $x, y, z \in H$ be such that $x \circ (y \circ z) \subseteq \bigcap_{I \in \mathscr{I}} I$ and $y \in \bigcap_{I \in \mathscr{I}} I$. Then $x \circ (y \circ z) \subseteq I$ and $y \in I$ for all $I \in \mathscr{I}$. Since I is a UP-hyperideal of type 1 of H, we have $x \circ y \subseteq I$ for all $I \in \mathscr{I}$ and so $x \circ y \subseteq \bigcap_{I \in \mathscr{I}} I$. Hence, $\bigcap_{I \in \mathscr{I}} I$ is a UP-hyperideal of type 1 of H. □

Remark 4.18. *The intersection of two UP-hyperideals of type 2 of a UP-hyperalgebra need not be a UP-hyperideal of type 2. We show the remark with Example 4.19.*

Example 4.19. *Let $H = \{0, 1, 2, 3\}$ be a set with a binary hyperoperation $\circ$ defined by the following Cayley table:*

$\circ$	0	1	2	3
0	$\{0\}$	$\{1\}$	$\{2,3\}$	$\{2,3\}$
1	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{1,2,3\}$
2	$\{0,2\}$	$\{2,3\}$	$\{0,2\}$	$\{1,2,3\}$
3	$\{0,3\}$	$\{1,2\}$	$\{0,1,3\}$	$\{2,3\}$

Then $(H, \circ, 0)$ is a UP-hyperalgebra and $\{0, 1, 2\}$ and $\{0, 1, 3\}$ are UP-hyperideals of type 2. Then $\{0, 1, 2\} \cap \{0, 1, 3\} = \{0, 1\}$. Since $0 \circ (1 \circ 2) = \{0, 1\} \subseteq \{0, 1\}$ and $1 \in \{0, 1\}$ but $0 \circ 2 = \{2, 3\} \nsubseteq \{0, 1\}$. Hence, $\{0, 1\}$ is not a UP-hyperideal of type 2 of H.

Remark 4.20. *The union of two UP-hyperideals of type 1 (resp., UP-hyperideals of type 2) of a UP-hyperalgebra need not be a hyper UP-ideal of type 1 (resp., hyper UP-ideal of type 2). We show the remark with Example 4.21.*

Example 4.21. *From Example 4.10, we have* $\{0,2\}$ *and* $\{0,3\}$ *are UP-hyperideals of type 1 of* H *and also UP-hyperideals of type 2. Then* $\{0,2\} \cup \{0,3\} = \{0,2,3\}$. *Since* $0 \circ (2 \circ 1) = \{2,3\} \subseteq \{0,2,3\}$ *and* $2 \in \{0,2,3\}$ *but* $(0 \circ 1) \cap \{0,2,3\} = \{1\} \cap \{0,2,3\} = \emptyset$, *we have* $\{0,2,3\}$ *is not a UP-hyperideal of type 2 of* H *and also not a UP-hyperideal of type 1.*

Remark 4.22. *The hyperproduct of two UP-hyperideals of type 1 (resp., UP-hyperideals of type 2) of a UP-hyperalgebra need not be a hyper UP-ideal of type 1 (resp., hyper UP-ideal of type 2). We show the remark with Example 4.23.*

Example 4.23. *From Example 4.21, we have* $\{0,3\}$ *is a UP-hyperideal of type 1 of* H *and also a UP-hyperideal of type 2. Then* $\{0,3\} \circ \{0,3\} = \{0,2,3\}$. *It follows from Example 4.21 that* $\{0,2,3\}$ *is not a UP-hyperideal of type 2 of* H *and also not a UP-hyperideal of type 1.*

Theorem 4.24. *Let* $\mathscr{I}$ *be a nonempty family of s-UP-hyperideals of type 1 of* H. *Then* $\bigcap_{I \in \mathscr{I}} I$ *is an s-UP-hyperideal of type 1 of* H.

Proof. Clearly, $0 \in I$ for all $I \in \mathscr{I}$. Then $0 \in \bigcap_{I \in \mathscr{I}} I$. Let $x, y, z \in H$ be such that $(x \circ (y \circ z)) \cap \bigcap_{I \in \mathscr{I}} I \neq \emptyset$ and $y \in \bigcap_{I \in \mathscr{I}} I$. Then $(x \circ (y \circ z)) \cap I \neq \emptyset$ and $y \in I$ for all $I \in \mathscr{I}$. Since I is an s-UP-hyperideal of type 1 of H, we have $x \circ y \subseteq I$ for all $I \in \mathscr{I}$ and so $x \circ y \subseteq \bigcap_{I \in \mathscr{I}} I$. Hence, $\bigcap_{I \in \mathscr{I}} I$ is an s-UP-hyperideal of type 1 of H. □

Remark 4.25. *The intersection of two s-UP-hyperideals of type 2 of a UP-hyperalgebra need not be an s-UP-hyperideal of type 2. We show the remark with Example 4.26.*

Example 4.26. *From Example 4.19, we have* $\{0,1,2\}$ *and* $\{0,1,3\}$ *are s-UP-hyperideals of type 2 of* H. *Then* $\{0,1,2\} \cap \{0,1,3\} = \{0,1\}$. *Since* $(0 \circ (1 \circ 3)) \cap \{0,1\} = \{1,2,3\} \cap \{0,1\} \neq \emptyset$ *and* $1 \in \{0,1\}$ *but* $(0 \circ 3) \cap \{0,1\} = \{2,3\} \cap \{0,1\} = \emptyset$, *we have* $\{0,1\}$ *is not an s-UP-hyperideal of type 2 of* H.

Remark 4.27. *The union of two s-UP-hyperideals of type 1 of a UP-hyperalgebra need not be an s-UP-hyperideal of type 1. We show the remark with Example 4.28.*

Example 4.28. *Let* $H = \{0,1,2,3\}$ *be a set with a binary hyperoperation* $\circ$ *defined by the following Cayley table:*

$\circ$	0	1	2	3
0	$\{0\}$	$\{1\}$	$\{2\}$	$\{3\}$
1	$\{0\}$	$\{0\}$	$\{2\}$	$\{2\}$
2	$\{0\}$	$\{1\}$	$\{0\}$	$\{1\}$
3	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$

Then $(H, \circ, 0)$ *is a UP-hyperalgebra and* $\{0,1\}$ *and* $\{0,2\}$ *are s-UP-hyperideals of type 1 and also s-UP-hyperideals of type 2. Then* $\{0,1\} \cap \{0,2\} = \{0,1,2\}$. *Since* $(0 \circ (1 \circ 3)) \cap \{0,1,2\} = \{2\} \cap \{0,1,2\} \neq \emptyset$ *and* $1 \in \{0,1,2\}$ *but* $(0 \circ 3) \cap \{0,1,2\} = \{3\} \cap \{0,1,2\} = \emptyset$. *Hence,* $\{0,1,2\}$ *is not an s-UP-hyperideal of type 2 of* H *and also not an s-UP-hyperideal of type 1.*

Remark 4.29. *The hyperproduct of two s-UP-hyperideals of type 2 of a UP-hyperalgebra need not be an s-UP-hyperideal of type 2. We show the remark with Example 4.30.*

Example 4.30. *Let $H = \{0,1,2,3\}$ be a set with a binary hyperoperation $\circ$ defined by the following Cayley table:*

$\circ$	0	1	2	3
0	$\{0,1\}$	$\{1\}$	$\{2\}$	$\{3\}$
1	$\{0\}$	$\{0\}$	$\{1,2\}$	$\{2\}$
2	$\{0\}$	$\{1,3\}$	$\{0\}$	$\{1\}$
3	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$

Then $(H,\circ,0)$ is a UP-hyperalgebra and $\{0\}$ is an s-UP-hyperideal of type 2. Then $\{0\}\circ\{0\} = \{0,1\}$. Since $(0\circ(1\circ 2))\cap\{0,1\} = \{1,2\}\cap\{0,1\} \neq \emptyset$ and $1 \in \{0,1\}$ but $(0\circ 2)\cap\{0,1\} = \{2\}\cap\{0,1\} = \emptyset$. Hence, $\{0,1\}$ is not an s-UP-hyperideal of type 2 of H.

Open Problem. Is the hyperproduct of two s-UP-hyperideals of type 1 of a UP-hyperalgebra an s-hyper UP-ideal of type 1?

By the definition of $\circ$ in Theorem 3.12 and Theorem 4.2, we have the following proposition.

Proposition 4.31. *Every UP-ideal of a UP-algebra H is a UP-hyperideal of type 1 (resp. s-UP-hyperideal of type 2, UP-hyperideal of type 1, UP-hyperideal of type 2) of the UP-hyperalgebra H, which is defined in Theorem 3.12.*

Example 4.32. *Let $H = \{0,1,2,3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	2
3	0	1	0	0

Then $(H,\cdot,0)$ is a UP-algebra and $S := \{0,2\}$ is a UP-subalgebra of H but it is not a UP-ideal of H. Indeed, $0\cdot(2\cdot 3) = 2 \in S$ and $2 \in S$, but $0\cdot 3 = 3 \notin S$. By Remark 3.23, we have S is a UP-hypersubalgebra of the UP-hyperalgebra H, which is defined in Theorem 3.12. Since $0\circ(2\circ 3) = 0\circ\{2\} = \{2\} \subseteq \{0,2\} = S$ and $2 \in S$, but $0\circ 3 = \{3\} \nsubseteq \{0,2\} = S$. Hence, S is not a UP-hyperideal of type 2 of H and also is not a UP-hyperideal of type 1.

Remark 4.33. *From Example 4.32, we have a UP-hypersubalgebra of H is not a UP-hyperideal of type 2 in general. Also, a UP-hypersubalgebra of H is not a UP-hyperideal of type 1 in general.*

By Remarks 4.11 and 4.33, Theorem 4.2 (3), and Example 4.5, we have that a UP-hyperideal of type 1 is not a UP-hypersubalgebra, a hyper UP-ideal of type 2 is not a UP-hypersubalgebra, a UP-hypersubalgebra is not a hyper UP-ideal of type 2, a UP-hypersubalgebra is not a hyper UP-ideal of type 1 in general, but the concept of hyper UP-ideals of type 2 is a generalization of hyper UP-ideals of type 1. Then, we get the diagram as shown in Figure 3.

Definition 4.34. *A nonempty subset I of H satisfies the closed condition if*

$$(\forall x,y \in H)(x \ll y, y \in I \Rightarrow x \in I).$$

Example 4.35. *From Example 4.10, we have $\{1\},\{3\},\{1,3\},\{1,2,3\}$, and H are all subsets of H satisfying the closed condition.*

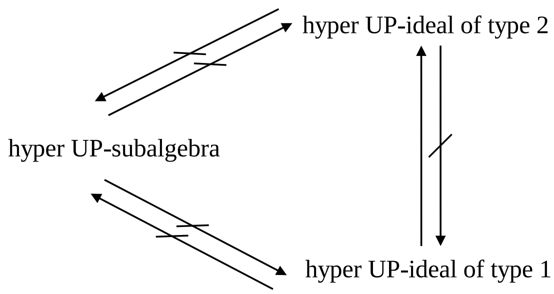

Figure 3: UP-hypersubalgebras and UP-hyperideals of types 1 and 2

Theorem 4.36. *Let $\mathscr{C}$ be a nonempty family of nonempty subsets of H satisfy the closed condition. Then $\bigcup_{C\in\mathscr{C}} C$ and $\bigcap_{C\in\mathscr{C}} C$ satisfy the closed condition if $\bigcap_{C\in\mathscr{C}} C$ is nonempty.*

Proof. Let $x, y \in H$ be such that $x \ll y$ and $y \in \bigcup_{C\in\mathscr{C}} C$. Then $y \in C$ for some $C \in \mathscr{C}$. Since C satisfies the closed condition, we have $x \in C \subseteq \bigcup_{C\in\mathscr{C}} C$. Hence, $\bigcup_{C\in\mathscr{C}} C$ satisfies the closed condition. Assume that $\bigcap_{C\in\mathscr{C}} C$ is nonempty. Let $x, y \in H$ be such that $x \ll y$ and $y \in \bigcap_{C\in\mathscr{C}} C$. Then $y \in C$ for all $C \in \mathscr{C}$. Since C satisfies the closed condition for all $C \in \mathscr{C}$, we have $x \in C$ for all $C \in \mathscr{C}$. Thus $x \in \bigcap_{C\in\mathscr{C}} C$. Hence, $\bigcap_{C\in\mathscr{C}} C$ satisfies the closed condition. □

Lemma 4.37. *If a nonempty subset I of H satisfies the closed condition, then for any $A \subseteq H$, $A \ll I$ implies $A \subseteq I$.*

Proof. Let $A \subseteq H$ be such that $A \ll I$ and let $x \in A$. Then $x \ll y$ for some $y \in I$. By the closed condition of I, we have $x \in I$. Hence, $A \subseteq I$. □

Theorem 4.38. *If a nonempty subset I of H containing 0 satisfies the closed condition, then $I = H$. Moreover, H is the only closed UP-hypersubalgebra (resp., closed UP-hyperideal of types 1 and 2, closed s-UP-hyperideal of types 1 and 2) of H.*

Proof. It is straightforward by Proposition 3.14 (10) and Lemma 4.37. □

The following proposition follows from Proposition 3.14 (11) and the definition of a UP-hyperideal of types 1 and 2.

Proposition 4.39. *Let A and B be subsets of H. Then the following statements hold:*

(1) if I is a UP-hyperideal of type 1 of H and if $A \circ (x \circ B) \subseteq I$ for $x \in I$, then $A \circ B \subseteq I$, and

(2) if I is a UP-hyperideal of type 2 of H and if $A \circ (x \circ B) \subseteq I$ for $x \in I$, then $(A \circ B) \cap I \neq \emptyset$.

Definition 4.40. *A nonempty subset I of H satisfies the R-condition if $0 \circ I = I$.*

Example 4.41. *From Example 4.10, we have $\{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}, \{2, 3\}$, and H are all subsets of H satisfying the R-condition.*

Theorem 4.42. *If I is a UP-hyperideal of type 1 (and also an s-UP-hyperideal of type 1) of H satisfying the R-condition, then*

$$(\forall a \in I, \forall x \in H)(a \circ x \subseteq I \Rightarrow x \in I).$$

Moreover,

$$(\forall a \in I, \forall A \subseteq H)(a \circ A \subseteq I \Rightarrow A \subseteq I).$$

Proof. Let $a \in I$ and $x \in H$ be such that $a \circ x \subseteq I$. By Proposition 3.14 (11) and the R-condition, we have $0 \circ (a \circ x) \subseteq 0 \circ I = I$. Since I is a UP-hyperideal of type 1 of H and by 3.2, we have $x \in 0 \circ x \subseteq I$. □

Theorem 4.43. *If I is an s-UP-hyperideal of type 1 of H satisfying the R-condition, then*

$$(\forall a \in I, \forall x \in H)((a \circ x) \cap I \neq \emptyset \Rightarrow x \in I).$$

Moreover,

$$(\forall A \subseteq H)((\forall x \in A, \exists a \in I)((a \circ x) \cap I \neq \emptyset) \Rightarrow A \subseteq I).$$

Proof. Let $a \in I$ and $x \in H$ be such that $(a \circ x) \cap I \neq \emptyset$. Then we choose an element $b \in (a \circ x) \cap I$. By Proposition 3.14 (7) and the R-condition, we have $b \in 0 \circ (a \circ x)$ and $b \in 0 \circ I = I$. Thus $(0 \circ (a \circ x)) \cap I \neq \emptyset$. Since I is an s-UP-hyperideal of type 1 of H and by 3.2, we have $x \in 0 \circ x \subseteq I$. □

5 Conclusions and future work

In this paper, we have introduced the concept of UP-hyperalgebras which is a generalization of UP-algebras, and investigated some related properties. Moreover, the concepts of UP-hypersubalgebras, UP-hyperideals of types 1 and 2, and s-UP-hyperideals of types 1 and 2 in UP-hyperalgebras are introduced and some relations among these concepts are presented.

In our future study of UP-hyperalgebras, may be the following topics should be considered:

(1) To get more results in UP-hyperalgebras and application.

(2) To study the fuzzy set theory of UP-hypersubalgebras, UP-hyperideals of types 1 and 2, and s-UP-hyperideal of types 1 and 2.

(3) To define Smarandache structure of UP-hyperalgebras.

(4) To get more connection between UP-hyperalgebras and Smarandache UP-hyperalgebras.

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