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On the equivalence of sequences dependent on fuzzy ideals in the BCI-algebra

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Abstract

Murali and Makamba (2001) introduced an equivalence of fuzzy subgroups. Dudek and Jun (2004) studied the equivalence defined by Murali and Makamba in fuzzy ideals of a BCI-algebra. In this paper, we obtained a sequence of fuzzy ideals of a BCI-algebra X from a fuzzy ideal on X. We will show that, if two fuzzy ideals are equivalent, then the sequence of fuzzy ideals obtained from them are equivalent. We show that there is a relationship between a fuzzy ideal with BCI-algebra X and a fuzzy ideal with adjoint BCI-algebra A, where A is an Abelian subgroup of $Aut_\mu(X)$.

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1 Introduction

Non-classical logic has become a considerable formal tool for computer science and artificial intelligence to deal with fuzzy information and uncertainty information. Many-valued logic, a great extension and development of classical logic, has always been a crucial direction in non-classical logic. Since 1965 Zadeh's [17] invention, the concept of fuzzy sets has been extensively applied to many mathematical field. On the other hand, the concept of BCI/BCK-algebras introduced by Iseki [4] and it has been raised by Imai and Iseki [3]. Xi [16] used the theory of fuzzy sets to BCK-algebras. Lee, Jun, Liu and several researchers investigated fuzzy ideals in BCI-algebras [7, 8, 9]. Jun (2011) studied fuzzy translations, fuzzy extensions and fuzzy multiplications of fuzzy sub BCI-algebras and ideals in BCK/BCI-algebras [5, 6]. They investigated relations among

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fuzzy translations, fuzzy extensions and fuzzy multiplications. In 2015, Senapati et al., [12, 13], introduced the notation of fuzzy translation of fuzzy H-ideals and also they studied intuitionistic fuzzy translation in BCI-algebras. Also, Senapati studied some applications of fuzzy translations in B-algebras [11, 14]. In 2016, Senapati et al., studied Atanassov's intuitionistic fuzzy bi-normed KU-ideals of a KU-algebra and they discussed some properties of it [15]. In 2004, Dudek and Jun studied the equivalence defined by Murali and Makamba [10] in fuzzy ideals in a BCI-algebra [1]. In this paper, we first show that from any fuzzy ideal, a sequence of fuzzy ideals can be obtained in a BCI-algebra and in the final part, the equivalences raised by Murali and Makamba and its generalization in fuzzy ideals of a BCI-algebra are studied. Also, we obtain a sequence fuzzy ideals of adjoint BCI-algebra A, where A is an Abelian subgroups of $AUT_{\sim}(X)$, from a fuzzy ideal on BCI-algebra X.

2 Some fuzzy ideals obtained from fuzzy ideal on a BCI-algebra

By a BCI-algebra we mean an algebra $(X; *, 0)$ of type $(2, 0)$ satisfying the following axioms for all $x, y \in X$:

(1) $((x * y) * (x * z)) * (z * y) = 0$,

(2) $(x * (x * y)) * y) = 0$,

(3) $x * x = 0$,

(4) $x * y = 0$ and $y * x = 0$ imply $x = y$.

for all $x, y, z \in X$. We can define a partial ordering $'' \leq ''$ on X by $x \leq y$ if and only if $x * y = 0$.

In this paper we consider that X is a BCI-algebra.

The following statements are true in any BCI-algebra X for all $x, y, z \in X$:

(1.1) $(x * y) * z = (x * z) * y$,

(1.2) $x * 0 = x$,

(1.3) $(x * z) * (y * z) \leq x * y$,

(1.4) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$,

(1.5) $0 * (x * y) = (0 * x) * (0 * y)$,

(1.6) $x * (x * (x * y)) = x * y$.

A non-empty subset I of X is called an ideal on X if it satisfies:

(I_1) $0 \in I$,

(I_2) $x * y \in I$ and $y \in I$ imply $x \in I$.

Moreover, a nonempty subset I of X is called a p-ideal on X if it satisfies condition (I_1) and

(I_3) $(x * z) * (y * z) \in I$ and $y \in I$ imply $x \in I$.

Putting $z = 0$ in (I_3), we can see that every p-ideal is an ideal.

A fuzzy subset on a set X is defined as a mapping $\mu : X \to [0, 1]$. Moreover, we define

$$\alpha_\mu = Sup\{\mu(x) \mid x \in X\}.$$

Definition 2.1. [1] *A fuzzy subset μ of BCI-algebra X is called a fuzzy ideal on X if it satisfies for all $x, y \in X$:*

(FI_1) $\mu(0) \geq \mu(x)$,

(FI_2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$.

Definition 2.2. [1] *A fuzzy subset μ in a BCI-algebra X is called a fuzzy p-ideal on X if satisfies condition (FI_1) and*

(FI_3) $\mu(x) \geq \min\{\mu((x * z) * (y * z)), \mu(y)\}, \forall x, y, z \in X$.

Any fuzzy p-ideal is a fuzzy ideal.

Definition 2.3. [1] *Let μ and ν be two fuzzy ideals on underlying of X and X', respectively. We say that μ and ν are strong equivalent and we write $\mu \approx \nu$, if there is a bijective function $f : X \longrightarrow X'$ such that for all $x, y \in X$:*

$$\mu \approx \nu \Longleftrightarrow \begin{cases} \mu(x) > \mu(y) \Leftrightarrow \nu(f(x)) > \nu(f(y)), \\ \mu(x) = 1 \Leftrightarrow \nu(f(x)) = 1, \\ \mu(x) = 0 \Leftrightarrow \nu(f(x)) = 0. \end{cases}$$

In Definition 2.3, if we subset $X = X'$ and $f = id_X$, then we have the next Definition:

Definition 2.4. [1] *Let μ and ν be two fuzzy ideals of X. We say that μ and ν are equivalent and we write $\mu \sim \nu$,*

$$\mu \sim \nu \Longleftrightarrow \begin{cases} \mu(x) > \mu(y) \Leftrightarrow \nu(x) > \nu(y), \\ \mu(x) = 1 \Leftrightarrow \nu(x) = 1, \\ \mu(x) = 0 \Leftrightarrow \nu(x) = 0. \end{cases}$$

for all $x, y \in X$.

Theorem 2.5. *Equivalency (strong equivalency) between fuzzy ideals of a BCI-algebra is an equivalence relation.*

Example 2.6. [1] *Let $X = \{0, 1, 2, 3\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3
0	0	0	2	2
1	1	0	3	2
2	2	2	0	0
3	3	2	1	0

Define fuzzy subsets μ and ν in X as follows:

$$\mu(x) = \begin{cases} 1 & for \quad x = 0 \\ 0.5 & for \quad x = 1 \\ 0.3 & for \quad x \in \{2, 3\}, \end{cases} \qquad \nu(x) = \begin{cases} 1 & for \quad x = 0 \\ 0.5 & for \quad x = 2 \\ 0.3 & for \quad x \in \{1, 3\} \end{cases}$$

Then μ and ν are not equivalent because $\mu(1) > \mu(2)$ but $\nu(1) \not> \nu(2)$.

Definition 2.7. *Let $(X; *, 0)$ and $(X'; *', 0')$ be two BCI-algebras. A mapping f from X to X' is called a BCI-homomorphism if*

$$f(x * y) = f(x) *' f(y) \text{ for all } x, y \in X.$$

BCI-homomorphism f is called a BCI-isomorphism if it is bijective.

Theorem 2.8. *Let μ and ν be two fuzzy subsets on underlying of X and X', respectively. Let $f : X \longrightarrow X'$ be a bijective map such that for all $x, y \in X$:*

(1) $f(x * y) = f(x) *' f(y)$ *for all* $x, y \in X$.

(2) $\mu(x) > \mu(y) \Leftrightarrow \nu(f(x)) > \nu(f(y))$,

(3) $\mu(x) = 1 \Leftrightarrow \nu(f(x)) = 1$,

(4) $\mu(x) = 0 \Leftrightarrow \nu(f(x)) = 0$.

If μ is a fuzzy ideal on BCI-algebra X, then ν is a fuzzy ideal on BCI-algebra X'.

Proof. Let $x', y' \in X'$, then there are $x, y \in X$ such that $\phi(x) = x'$ and $\phi(y) = y'$. Since μ is a fuzzy ideal, we get $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$. So, $\mu(x) \geq \mu(x * y)$ or $\mu(x) \geq \mu(y)$. Therefore,

$$\nu(\phi(x)) \geq \nu(\phi(x * y)) \text{ or } \nu(\phi(x)) \geq \nu(\phi(y)),$$

and so, $\nu(x') \geq \min\{\nu(x' *' y'), \nu(y')\}$.

Moreover, $\mu(0) \geq \mu(x)$ implies $\nu(0') \geq \nu(x')$ and so ν is a fuzzy ideal on X'. □

Theorem 2.9. *Let μ be a fuzzy ideal on finite BCI-algebra X and $f : X \longrightarrow X$ be a function such that for all $x, y \in X$. Then $\mu(x) > \mu(y) \Leftrightarrow \mu(f(x)) > \mu(f(y))$. Also, for all $x \in X$ we have $\mu(f(x)) = \mu(x))$.*

Proof. Let $\mu(f(x)) > \mu(x))$ for some $x \in X$. Then

$$\mu(f^n(x)) > \ldots > \mu(f(x)) > \mu(x)),$$

and it contradicts the fact that X is finite. □

Group-like BCI-algebras are described in [1]. Moreover, in Example 1.1.2 [2] BCI-algebra obtained from an Abelian group.

Example 2.10. [2] *Suppose $(X; \cdot, 0)$ is an Abelian group with 0 as the identity element. Define a binary operation $*$ on X by putting $x * y = x \cdot y^{-1}$. Then $(X; *, 0)$ is a BCI-algebra.*

We call $(X; *, 0)$ in the above example the adjoint BCI-algebra of the Abelian group $(X; \cdot, 0)$.

Theorem 2.11. *Let μ be a fuzzy subset in X and $\mathrm{Im}(\mu) = \{\lambda_0, \lambda_1, \ldots, \lambda_n\}$, where $\lambda_0 > \lambda_1 > \ldots > \lambda_n$. If $X_0 \subset X_1 \subset \ldots \subset X_n = X$ are p-ideals of X such that $\mu(X_k \backslash X_{k-1}) = \lambda_k$ for $k = 0, 1, \ldots, n$, where $X_{-1} = \emptyset$, then μ is a fuzzy p-ideal in X.*

Example 2.12. *Let $(X; *, 0)$ be an adjoint BCI-algebra of the Abelian group $(\mathbb{Z}_4; +, 0)$. Then $x * y = (x + 3y)(\mathrm{mod}4)$ and $\emptyset \subset X_1 \subset X_2 \subset X_3$, where $X_1 = \{0\}$, $X_2 = \{0, 2\} \simeq \mathbb{Z}_2, X_3 = \mathbb{Z}_4$, is $1 > \alpha > \beta > 0$.*

$$\mu(x) = \begin{cases} 1 & \textit{for} \quad x \in X_1 \\ \alpha & \textit{for} \quad x \in X_2 \backslash X_1 \\ \beta & \textit{for} \quad x \in X_3 \backslash X_2 \end{cases}$$

Then μ is a fuzzy p-ideal on $(X; *, 0)$.

Let μ be a fuzzy ideal on BCI-algebra $(X; *, 0)$. We define

$$Aut_\mu(X) = \{f : X \to X | f \text{ is a bijective map and } \mu(f(x)) = \mu(x), \forall x \in X\}.$$

Lemma 2.13. *Let μ be a fuzzy ideal on BCI-algebra $(X;*,0)$. Define a binary operation $\circ$ on $Aut_\mu(X)$ by putting $f\circ g(x) = f(g(x))$. Then $(Aut_\mu(X);\circ, id_X)$ is a group.*

Proposition 2.14. *Let μ be a fuzzy subset on BCI-algebra $(X;*,0)$. Let A be an Abelian subgroup of $(Aut_\mu(X);\circ, id_X)$. Then $(A;\star, id_X)$ is a BCI-algebra when $f\star g = f\circ g^{-1}$. In fact, $(A;\star, id_X)$ is the adjoint BCI-algebra of the Abelian group $(A;\circ, id_X)$.*

Proof. It obtains from Example 2.10. □

Proposition 2.15. *Let μ be a fuzzy ideal on BCI-algebra $(X;*,0)$ and A be an Abelian subgroup of $(Aut_\mu(X);\circ, id_X)$. For all $f\in A$ and $n\geq 1$ define a map $\tau : A \longrightarrow [0,1]$ as follows*

$$\tau(f) = \begin{cases} \frac{\alpha_\mu}{1+\alpha_\mu} & f = id \\ \frac{\alpha_\mu}{n+\alpha_\mu} & f\in A-\{id\} \end{cases} \tag{1}$$

for all $f\in A$. Then τ is a fuzzy ideal on adjoint BCI-algebra $(A;\star, id_X)$.

Proof. By Theorem 2.11, clearly τ is a p-fuzzy ideal on A. Therefore, τ is a fuzzy ideal on A. □

Theorem 2.16. *Let A be an Abelian subgroup $(Aut_\mu(X);\circ, id_X)$. and $(id_X) = A_0 \subset A_1 \subset \ldots \subset A_n = A$ be a maximal chain of ideals of adjoint BCI-algebra $(A;\star, id_X)$. For every $\lambda_i\in[0,1], i\in\{1,\ldots,n\}, 1\geq\lambda_1\geq\ldots\geq\lambda_n$, define μ as follows:*

$$\mu(x) = \begin{cases} 1 & x\in A_0 \\ \lambda_1 & x\in A_1 - A_0 \\ \lambda_2 & x\in A_2 - A_1 \\ \vdots & \vdots \\ \lambda_n & x\in A_n - A_{n-1} \end{cases}$$

Then μ is a fuzzy ideal on A.

Proposition 2.17. *Let μ be a fuzzy ideal on BCI-algebra X and A be an Abelian subgroup of $(Aut_\mu(X);\circ, id_X)$. For all $f\in A$, for $a,b>0$, define a map $\varrho : A\longrightarrow [0,1]$ as follows*

$$\varrho(f) = \begin{cases} \frac{a}{b+a} & f \text{ is an even permutation} \\ 0 & f \text{ is an odd permutation,} \end{cases} \tag{2}$$

for all $f\in A$. Then ϱ is a fuzzy ideal on adjoint BCI-algebra $(A;\star, id_X)$.

Proof. We prove this result in two following cases:

Case 1) Suppose that $f,g\in A$ both are even or odd permutation. Hence, $f\star g$ becomes an even permutation. If f and g both are even permutation, then

$$\begin{aligned} \varrho(f) &= \frac{a}{b+a} \\ &\geq \min\left\{\frac{a}{b+a}, \frac{a}{b+a}\right\} \\ &= \min\{\varrho(f\star g), \varrho(g)\}. \end{aligned}$$

If f and g both are odd permutation, then

$$\begin{aligned}\varrho(f) &= 0\\ &\geq \min\{0, \frac{a}{b+a}\}\\ &= \min\{\varrho(f\star g), \varrho(g)\}.\end{aligned}$$

Case 2) Suppose that $f, g \in A$ such that one of them is even permutation and the other one is odd permutation. Thus $f \star g$ becomes an odd permutation. First, consider f is odd, then

$$0 = \varrho(f) = \min\left\{0, \frac{a}{b+a}\right\} = \min\{\varrho(f\star g), \varrho(g)\}.$$

Else

$$\frac{a}{b+a} = \varrho(f) \geq \min\{0, 0\} = \min\{\varrho(f\star g), \varrho(g)\}.$$

Therefore, ϱ is a fuzzy ideal on A. □

Theorem 2.18. *Let μ be a fuzzy ideal on BCI-algebra X. If for all $x \in X$ and $i \in \mathbb{N}_n$,*

$$\mu_1(x) = \mu(x) \quad \textit{and} \quad \mu_i(x) = \frac{\mu_{i-1}(x)}{1+\alpha_{\mu_{i-1}} - \mu_{i-1}(x)}, \quad \textit{for} \quad i \geq 2.$$

Then $\{\mu_i\}_{i\in\mathbb{N}}$ is a sequence of fuzzy ideals of X.

Proof. It is easy to see that for all $x \in X$ and $n \geq 2$

$$0 \leq \frac{\mu_{n-1}(x)}{1+\alpha_{\mu_{n-1}} - \mu_{n-1}(x)} \leq 1.$$

Now, suppose that x and y are elements of X. If $\mu(x) \geq \mu(xy)$, for all $x, y \in G$, then

$$\begin{aligned}\mu_2(x) &= \frac{\mu_1(x)}{1+\alpha_{\mu_1} - \mu_1(x)}\\ &\geq \frac{\mu_1(xy)}{1+\alpha_{\mu_1} - \mu_1(xy)} = \mu_2(xy)\\ &\geq \mu_2(xy) \wedge \mu_2(y).\end{aligned}$$

Similarly, if $\mu(x) \geq \mu(y)$, for all $x, y \in G$, one can show that $\mu_2(x) \geq \mu_2(y) \geq \mu_2(xy) \wedge \mu_2(y)$. Therefore, μ_2 is a fuzzy ideal on X.

By the similar way, we obtain for every n, μ_n is a fuzzy ideal on X. □

Theorem 2.19. *Let μ be a fuzzy ideal on BCI-algebra X. If for all $x \in X$ and $i, j \in \mathbb{N}_n$,*

$$\mu_{ij}(x) = \frac{\mu_j(x)}{\max\left\{\alpha_{\mu_i}, \alpha_{\mu_j}\right\} + \alpha_{\mu_j} - \mu_j(x)}.$$

Then $\{\mu_{ij}\}_{ij\in\mathbb{N}}$ is a sequence of fuzzy ideals of a BCI-algebra X.

Proof. It is easy to see that for all $x \in G,\ n \geq 2$, we have

$$0 \leq \frac{\mu_j(x)}{\max\{\alpha_{\mu_i}, \alpha_{\mu_j}\} + \alpha_{\mu_j} - \mu_j(x)} \leq 1.$$

Now, suppose x and y are elements of X. If $\mu_j(x) \geq \mu_j(xy)$, then

$$\max\{\alpha_{\mu_i}, \alpha_{\mu_j}\} + \alpha_{\mu_j} - \mu_j(x) \leq \max\{\alpha_{\mu_i}, \alpha_{\mu_j}\} + \alpha_{\mu_j} - \mu_j(xy).$$

So, we have for all $x \in G$,

$$\frac{\mu_j(x)}{\max\{\alpha_{\mu_i}, \alpha_{\mu_j}\} + \alpha_{\mu_j} - \mu_j(x)} \geq \frac{\mu_j(xy)}{\max\{\alpha_{\mu_i}, \alpha_{\mu_j}\} + \alpha_{\mu_j} - \mu_j(x)}.$$

Thus,

$$\mu_{ij}(x) \geq \mu_{ij}(xy) \geq \mu_{ij}(xy) \wedge \mu_{ij}(y).$$

Hence, μ_{ij} is a fuzzy ideal on X. □

3 On the equivalence of sequences on fuzzy ideals

Definition 3.1. *Let $\{\mu_i\}_{i\in\mathbb{N}}$ and $\{\mu'_i\}_{i\in\mathbb{N}}$ be two sequence of fuzzy ideals on X and X', respectively. We say that $\{\mu_i\}_{i\in\mathbb{N}}$ and $\{\mu'_i\}_{i\in\mathbb{N}}$ are strong equivalent, if $\mu_i \approx \nu_i$, for $i \in \mathbb{N}$.*

Moreover, if $\{\mu_i\}_{i\in\mathbb{N}}$ and $\{\mu'_i\}_{i\in\mathbb{N}}$ are two sequence of fuzzy ideals on X, then we say that $\{\mu_i\}_{i\in\mathbb{N}}$ and $\{\mu'_i\}_{i\in\mathbb{N}}$ are equivalent, if $\mu_i \sim \nu_i$, for $i \in \mathbb{N}$.

Moreover, $\{\mu_i\}_{i\in\mathbb{N}}$ is two sequence of fuzzy ideals on BCI-algebra $(X; *, 0)$. We define

$$Aut_{\{\mu_i\}_{i\in\mathbb{N}}}(X) = \{f : X \to X | f \in Aut_{\mu_i}, \forall i \in \mathbb{N}\}.$$

Lemma 3.2. *Let μ be a fuzzy ideal on BCI-algebra $(X; *, 0)$. Define a binary operation $\circ$ on $Aut_{\{\mu_i\}_{i\in\mathbb{N}}}(X)$ by putting $f \circ g(x) = f(g(x))$. Then $(Aut_{\{\mu_i\}_{i\in\mathbb{N}}}(X); \circ, id_X)$ is a group.*

In Theorem 2.18 we obtain a sequence of fuzzy ideals of a fuzzy ideal μ on BCI-algebra X. Now, we have:

Corollary 3.3. *Let μ and ν be two strong equivalent fuzzy ideals of X and X', respectively. Then $\{\mu_i\}_{i\in\mathbb{N}}$ and $\{\nu_i\}_{i\in\mathbb{N}}$ (Theorem 2.18) are strong equivalent fuzzy ideals on X and X', respectively.*

Proof. We have

$$\mu_i(x) > \mu_i(y) \Leftrightarrow \mu(x) > \mu(y) \Leftrightarrow \nu(f(x)) > \nu(f(y)) \Leftrightarrow \nu_i(f(x)) > \nu_i(f(y)).$$

This shows that proof is complete. □

By the similar way, we have

Corollary 3.4. *Let μ and ν be two equivalent fuzzy ideals of X. Then $\{\mu_i\}_{i\in\mathbb{N}}$ and $\{\nu_i\}_{i\in\mathbb{N}}$ (Theorem 2.18) are equivalent fuzzy ideals on X.*

In Theorem 2.19 we obtain a sequence of fuzzy ideals $\{\mu_{ij}\}_{i,j\in\mathbb{N}}$ of a fuzzy ideal μ on BCI-algebra X. Now, we have:

Corollary 3.5. *Let μ and ν be two (strong) equivalent fuzzy ideals of X and X', respectively. Then $\{\mu_{ij}\}_{i,j\in\mathbb{N}}$ and $\{\nu_{ij}\}_{i,j\in\mathbb{N}}$(Theorem 2.19) are equivalent fuzzy ideals of X and X', respectively.*

Corollary 3.6. *Let $\{\mu_i\}_{i\in\mathbb{N}}$ be a sequence of fuzzy ideals of BCI-algebra X and A be an Abelian subgroup of $(Aut_{\{\mu_i\}_{i\in\mathbb{N}}}(X);\circ, id_X)$. For all $f\in\ A$, define a map $\tau_i : A\longrightarrow[0,1]$ as follows*

$$\tau_i(f)=\begin{cases}\frac{\alpha_{\mu_i}}{i+\alpha_{\mu_i}} & f=id\\ \frac{\alpha_{\mu_i}}{n+\alpha_{\mu_i}} & f\in A-\{id\}\end{cases}\tag{3}$$

for all $f\in A$. $\{\tau_i\}_{i\in\mathbb{N}}$ is a sequence of fuzzy ideals of adjoint BCI-algebra A.

Proof. By Proposition 2.15, τ_i, $i\in\mathbb{N}$ is a fuzzy ideal on adjoin BCI-algebra A. Therefore $\{\tau_i\}_{i\in\mathbb{N}}$ is a sequence of fuzzy ideals of adjoint BCI-algebra A. □

Corollary 3.7. *Let $\{\mu_i\}_{i\in\mathbb{N}}$ be a sequence of fuzzy ideals of BCI-algebra X and A be an Abelian subgroup of $(Aut_{\{\mu_i\}_{i\in\mathbb{N}}}(X);\circ, id_X)$. For all $f\in A$, define a map $\varrho_i : A\longrightarrow[0,1]$ as follows*

$$\varrho_i(f)=\begin{cases}\frac{\alpha_{\mu_i}}{i+\alpha_{\mu_i}} & f \text{ is an even permutation}\\ 0 & f \text{ is an odd permutation,}\end{cases}\tag{4}$$

for all $f\in A$. Then $\{\varrho_i\}_{i\in\mathbb{N}}$ is a sequence of fuzzy ideals of adjoint BCI-algebra A.

Proof. By Proposition 2.17, ϱ_i, $i\in\mathbb{N}$ is a fuzzy ideal on adjoin BCI-algebra A. Therefore $\{\varrho_i\}_{i\in\mathbb{N}}$ is a sequence of fuzzy ideals of adjoint BCI-algebra A. □

Example 3.8. *Let $X=(\mathbb{Z}_4,-,0)$ be adjoint BCI-algebra of Abelian group $(\mathbb{Z}_4,+,0)$ (See Example 2.12). Define fuzzy subsets μ_1, μ_2 and μ_3 in X as follows:*

$$\mu_1(x)=\begin{cases}\frac{1}{2}, & x=0,2\\ \frac{1}{5}, & x=1,3,\end{cases}\qquad \mu_2(x)=\begin{cases}\frac{1}{2}, & x=0,2\\ \frac{2}{13}, & x=1,3,\end{cases}\qquad \mu_3(x)=\begin{cases}\frac{1}{2}, & x=0,2\\ \frac{4}{35}, & x=1,3.\end{cases}$$

It is not difficult to see that $Aut_{\{\mu_i\}_{i\in\mathbb{N}}}(X)=\{id,(02),(13),(02)(13)\}\cong K_4$. By Corollary 3.6, for i=1,2,3, we have

$$\tau_i(f)=\begin{cases}\frac{0.5}{i+0.5} & f=id\\ \frac{0.5}{3+0.5} & f\in A-\{id\}\end{cases}$$

and by Corollary 3.7, for i=1,2,3, we have

$$\varrho_i(f)=\begin{cases}\frac{0.5}{i+0.5} & f \text{ is an even permutation}\\ 0 & f \text{ is an odd permutation,}\end{cases}$$

Now, τ_i and ϱ_i are fuzzy ideal of adjoin BCI-algebra $Aut_{\{\mu_i\}_{i\in\mathbb{N}}}(X)$.

References

[1] W.A. Dudek, Y.B. Jun, *Quasi p-ideals of quasi BCI-algebras*, Quasigroups and Related Systems, 11 (2004), 25–38.

[2] Y. Huang, *BCI-Algebras*, Science Press, Beijing, 2006.

[3] Y. Imai, K. Iseki, *On axiom system of propositional calculi*, Proceedings of the Japan Academy, 42 (1966), 19–22.

[4] K. Iseki,*An algebra related with a propositional calculus*, Proceedings of the Japan Academy, 42 (1966), 26-29.

[5] Y.B. Jun, *Closed fuzzy ideals in BCI-algebras*, Mathematica Japonica, 38 (1993), 199–202.

[6] Y.B. Jun, *Translations of fuzzy ideals in BCK/BCI-algebras*, Hacettepe Journal of Mathematics and Statistics, 40 (2011), 349–358.

[7] K.J. Lee, Y.B. Jun, M.I. Doh,*Fuzzy translations and fuzzy multiplications of BCK/BCI-algebras*, Communications of the Korean Mathematical Society, 24 (2009), 353–360.

[8] Y.L. Liu, X.H. Zhang, *Fuzzy a-ideals in BCI-algebras*, Advances in Mathematics(China), 31 (2002), 65–73.

[9] Y.L. Liu, J. Meng, X.H. Zhang, Z.C. Yue, *q-ideals and a-ideals in BCI-algebras*, Southeast Asian Bulletin of Mathematics, 24 (2000), 243–253.

[10] V. Murali, B.B. Makamba, *On an equivalence of fuzzy subgroups I*, Fuzzy Sets and Systems, 123 (2001), 259–264.

[11] T. Senapati, *Translations of intuitionistic fuzzy B-algebras*, Fuzzy Information and Engineering, 7(4) (2015), 389–404.

[12] T. Senapati, M. Bhowmik, M. Pal, B. Davvaz, *Fuzzy translations of fuzzy H-ideals in BCK/BCI-algebras*, Journal of the Indonesian Mathematical Society, 21(1) (2015), 45–58.

[13] T. Senapati, M. Bhowmik, M. Pal, B. Davvaz, *Atanassov's intuitionistic fuzzy translation of intuitionistic fuzzy subalgebras and ideals in BCK/BCI-algebras*, Eurasian Mathematical Journal, 6(1) (2015), 96–114.

[14] T. Senapati, C.S. Kim, M. Bhowmik, M. Pal, *Cubic subalgebras and cubic closed ideals of B-algebras*, Fuzzy Information and Engineering, 7(2) (2015), 129–149.

[15] T. Senapati, K.P. Shum, *Atanassov's intuitionistic fuzzy bi-normed KU-ideals of a KU-algebra*, Journal of Intelligent and Fuzzy Systems, 30 (2016), 1169–1180.

[16] O. Xi, *Fuzzy BCK-algebras*, Mathematica Japonica, 36 (1991), 935–942.

[17] L.A. Zadeh, *Fuzzy sets*, Information and Control, 8 (1965), 338–353.