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Implicative ideals of BCK-algebras based on Dokdo structure

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Abstract

A Dokdo structure is used to study implicative ideals in BCK-algebras. The notion of Dokdo implicative ideals in BCK-algebras is introduced, and the relevant properties are investigated. The relationship between Dokdo subalgebras, Dokdo ideals, and Dokdo implicative ideals are discussed, and conditions that allow a Dokdo ideal and a Dokdo subalgebra to be a Dokdo implicative ideal are provided.

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1 Introduction

It is already well known that the fuzzy set introduced by Zade is an extension of the existing set using fuzzy logic, and has a wide range of applications. Fuzzy sets have been widely generalized by various methods such as bipolar fuzzy set, interval-valued fuzzy set, soft set, and so on. The bipolar fuzzy set, an extension of the fuzzy set, is a very useful tool for simultaneously processing negative and positive information, and the interval value fuzzy set is also an extension of the fuzzy set whose membership degree range is a subinterval of $[0, 1]$. Soft set theory is a good mathematical tool for dealing with uncertainty in a parametric manner and has useful applications for decision making, medical diagnosis, etc. Using the idea of Łukasiewicz t-norm, Jun [6] constructed the concept of Łukasiewicz fuzzy sets based on a given fuzzy set and applied it to BCK-algebras and BCI-algebras. Also, Song and Jun [14] used Łukasiewicz fuzzy sets to study positive implicative ideals in BCK-algebras. Jun et al. [7] introduced the notion of (inner, outer) crossing cubic structures which is an extension of bipolar-valued fuzzy sets, and investigated several properties. They defined the same direction order and the opposite direction order in crossing cubic structures. Jun and Song [8] used crossing cubic structures to study ideals in BCK/BCI-algebras. As we enter the information age, problems to be solved by utilizing hybrid structures in various fields are emerging. It is a natural phenomenon that the development of mathematical tools that can meet these needs must be made. To respond to this background and need, Jun [5] introduced a new type of hybrid structure called Dokdo structure, where "Dokdo" is the name of the most beautiful island in Korea, using the concepts of bipolar

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fuzzy set, soft set and interval-valued fuzzy and first applied it to the algebraic structure BCK/BCI-algebras (see [5]). As part of meeting this background and needs, Jun [5] introduced a new type of hybrid structure called the Dokdo structure using the concept of bipolar fuzzy sets, soft sets, and interval-value fuzzy sets, and first applied it to algebraic structures called BCK/BCI-algebra.

The purpose of this paper is to study implicative ideals in BCK-algebras using Dokdo structure. We introduce the concept of Dokdo implicative ideals in BCK-algebras and investigate the relevant properties. We discuss the relationship between Dokdo subalgebra, Dokdo ideal, and Dokdo implicative ideal. We provide conditions that allow Dokdo ideal and Dokdo subalgebra to be Dokdo implicative ideal.

2 Preliminaries

2.1 Basic concepts about BCK-algebra

BCI/BCK-algebra is an important type of logical algebra introduced by K. Iséki (see [3] and [4]), and it has been extensively investigated by several researchers. See the books [2, 12] for further information regarding BCI-algebras and BCK-algebras.

In this section, we recall the definitions and basic results required in this paper.

Let X be a set with a special element "0" and a binary operation " $*$ ". If it satisfies the following conditions:

(I1) $(\forall a, b, c \in X)\ (((a * b) * (a * c)) * (c * b) = 0)$,

(I2) $(\forall a, b \in X)\ ((a * (a * b)) * b = 0)$,

(I3) $(\forall a \in X)\ (a * a = 0)$,

(I4) $(\forall a, b \in X)\ (a * b = 0,\ b * a = 0\ \Rightarrow\ a = b)$,

(K) $(\forall a \in X)\ (0 * a = 0)$,

then it is called a *BCK-algebra*, and it is denoted by $(X, *, 0)$.

The order relation " $\leq$ " in a BCK-algebra $(X, *, 0)$ is defined as follows:

$$(\forall a, b \in X)(a \leq b\ \Leftrightarrow\ a * b = 0). \tag{1}$$

Every BCK-algebra $(X, *, 0)$ satisfies the following conditions (see [11, 12]):

$$(\forall a \in X)\,(a * 0 = a)\,, \tag{2}$$
$$(\forall a, b, c \in X)\,(a \leq b\ \Rightarrow\ a * c \leq b * c,\ c * b \leq c * a)\,, \tag{3}$$
$$(\forall a, b, c \in X)\,((a * b) * c = (a * c) * b)\,. \tag{4}$$

A BCK-algebra $(X, *, 0)$ is said to be *implicative* (see [12]) if $a * (b * a) = a$ for all $a, b \in X$.

A subset A of a BCK-algebra $(X, *, 0)$ is called

- a *subalgebra* of $(X, *, 0)$ (see [2, 12]) if it satisfies:

$$(\forall a, b \in A)(a * b \in A), \tag{5}$$

- an *ideal* of $(X, *, 0)$ (see [2, 12]) if it satisfies:

$$0 \in A, \tag{6}$$
$$(\forall a, b \in X)(a * b \in A,\ b \in A\ \Rightarrow\ a \in A). \tag{7}$$

A subset A of a BCK-algebra $(X, *, 0)$ is called an *implicative ideal* of $(X, *, 0)$ (see [12]) if it satisfies (6) and

$$(\forall a, b, c \in X)((a * (b * a)) * c \in A,\ c \in A\ \Rightarrow\ a \in A). \tag{8}$$

2.2 Basic concepts about Dokdo structure

Let X be a set. A *bipolar fuzzy set* in X (see [9]) is an object of the following type

$$\mathring{f} = \{(a, \mathring{f}^-(a), \mathring{f}^+(a)) \mid a \in X\}, \tag{9}$$

where $\mathring{f}^- : X \to [-1, 0]$ and $\mathring{f}^+ : X \to [0, 1]$ are mappings. The bipolar fuzzy set which is described in (9) is simply denoted by $\mathring{f} := (X; \mathring{f}^-, \mathring{f}^+)$.

A bipolar fuzzy set can be reinterpreted as a function:

$$\mathring{f} : X \to [-1, 0] \times [0, 1],\ a \mapsto (\mathring{f}^-(a), \mathring{f}^+(a)).$$

Denote by $BF(X)$ the set of all bipolar fuzzy sets in X. We define a binary relation "$\leq_b$" on $BF(X)$ as follows:

$$(\forall \mathring{f}, \mathring{g} \in BF(X)) \left(\mathring{f} \leq_b \mathring{g} \ \Leftrightarrow \left\{ \begin{array}{l} \mathring{f}^-(a) \geq \mathring{g}^-(a) \\ \mathring{f}^+(a) \leq \mathring{g}^+(a) \end{array} \right. \quad \text{for all } a \in X \right). \tag{10}$$

Then $(\mathrm{BF(X)}, \leq_b)$ is a poset.

Let U be an initial universe set and X be a set of parameters. For any subset A of X, a pair (f^s, A) is called a *soft set* over U (see [10, 13]), where f^s is a mapping described as follows:

$$f^s : A \to 2^U,$$

where 2^U is the power set of U. If $A = X$, the soft set (f^s, A) over U is simply denoted by f^s only.

A mapping $\tilde{f} : X \to [[0, 1]]$ is called an *interval-valued fuzzy set* (briefly, an IVF set) in X (see [1, 15]) where $[[0, 1]]$ is the set of all closed subintervals of $[0, 1]$, and members of $[[0, 1]]$ are called *interval numbers* and are denoted by $\tilde{a}$, $\tilde{b}$, $\tilde{c}$, etc., where $\tilde{a} = [a_l, a_r]$ with $0 \leq a_l \leq a_r \leq 1$.

For every two interval numbers $\tilde{a}$ and $\tilde{b}$, we define

$$\tilde{a} \trianglelefteq \tilde{b} \text{ (or } \tilde{b} \trianglerighteq \tilde{a}) \ \Leftrightarrow \ a_l \leq b_l,\ a_r \leq b_r, \tag{11}$$

$$\tilde{a} = \tilde{b} \ \Leftrightarrow \ \tilde{a} \trianglelefteq \tilde{b},\ \tilde{b} \trianglelefteq \tilde{a}, \tag{12}$$

$$\mathrm{rmin}\{\tilde{a}, \tilde{b}\} = [\min\{a_l, b_l\}, \min\{a_r, b_r\}]. \tag{13}$$

Let U be an initial universe set and X a set of parameters. A triple $Dok_f := (\mathring{f}, f^s, \tilde{f})$ is called a *Dokdo structure* (see [5]) in (U, X) if $\mathring{f} : X \to [-1, 0] \times [0, 1]$ is a bipolar fuzzy set in X, $f^s : X \to 2^U$ is a soft set over U and $\tilde{f} : X \to [[0, 1]]$ is an interval-valued fuzzy set in X.

The Dokdo structure $Dok_f := (\mathring{f}, f^s, \tilde{f})$ in (U, X) can be represented as follows:

$$\begin{array}{c} Dok_f := (\mathring{f}, f^s, \tilde{f}) : X \to ([-1, 0] \times [0, 1]) \times 2^U \times [[0, 1]], \\ x \mapsto \left(\mathring{f}(x), f^s(x), \tilde{f}(x)\right) \end{array} \tag{14}$$

where $\mathring{f}(x) = (\mathring{f}^-(x), \mathring{f}^+(x))$ and $\tilde{f}(x) = [\tilde{f}_L(x), \tilde{f}_R(x)]$.

Given a Dokdo structure $Dok_f := (\mathring{f}, f^s, \tilde{f})$ in a Dokdo universe (U, X), we consider the following sets:

$$\mathring{f}(\max, \min) := \left\{ \tfrac{x}{(y,z)} \in \tfrac{X}{X \times X} \,\middle|\, \begin{array}{l} \mathring{f}^-(x) \leq \max\{\mathring{f}^-(y), \mathring{f}^-(z)\} \\ \mathring{f}^+(x) \geq \min\{\mathring{f}^+(y), \mathring{f}^+(z)\} \end{array} \right\},$$

and

$$\begin{aligned} &\mathring{f}(t^-) := \{x \in X \mid \mathring{f}^-(x) \leq t^-\}, \\ &\mathring{f}(t^+) := \{x \in X \mid \mathring{f}^+(x) \geq t^+\}, \\ &\mathring{f}(t^-, t^+) := \mathring{f}(t^-) \cap \mathring{f}(t^+), \\ &f^s_\alpha := \{x \in X \mid f^s(x) \supseteq \alpha\}, \\ &\tilde{f}_{\tilde{a}} := \{x \in X \mid \tilde{f}(x) \trianglerighteq \tilde{a}\}, \end{aligned}$$

where $(t^-, t^+) \in [-1, 0] \times [0, 1]$, $\alpha \in 2^U$ and $\tilde{a} = [a_l, a_r]$.

In what follows, let U be an initial universe set and X a set of parameters unless otherwise specified. We say that the pair (U, X) is a BCK-Dokdo universe if $(X, *, 0)$ is a BCK-algebra.

Definition 2.1 ([5])**.** *Let (U, X) be a BCK-Dokdo universe. A Dokdo structure $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ in (U, X) is called a Dokdo subalgebra of a BCK-algebra $(X, *, 0)$ if it satisfies:*

$$(\forall x, y \in X)\left(\tfrac{x*y}{(x,y)} \in \mathring{f}(\max, \min)\right), \tag{15}$$

$$(\forall x, y \in X)\left(f^s(x*y) \supseteq f^s(x) \cap f^s(y)\right), \tag{16}$$

$$(\forall x, y \in X)\left(\tilde{f}(x*y) \trianglerighteq \mathrm{rmin}\{\tilde{f}(x), \tilde{f}(y)\}\right). \tag{17}$$

Definition 2.2. *Let (U, X) be a BCK-Dokdo universe. A Dokdo structure $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ in (U, X) is called a Dokdo ideal of a BCK-algebra $(X, *, 0)$ (see [5]) if it satisfies:*

$$(\forall x \in X)\left(\begin{array}{l} \frac{0}{(x,x)} \in \mathring{f}(\max, \min), \\ f^s(0) \supseteq f^s(x), \\ \tilde{f}(0) \trianglerighteq \tilde{f}(x) \end{array}\right). \tag{18}$$

$$(\forall x, y \in X)\left(\begin{array}{l} \frac{x}{(x*y,\, y)} \in \mathring{f}(\max, \min), \\ f^s(x) \supseteq f^s(x*y) \cap f^s(y), \\ \tilde{f}(x) \trianglerighteq \mathrm{rmin}\{\tilde{f}(x*y), \tilde{f}(y)\} \end{array}\right). \tag{19}$$

Lemma 2.3 ([5])**.** *Every Dokdo ideal $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ of a BCK-algebra $(X, *, 0)$ satisfies:*

$$(\forall x, y \in X)\left(x \le y \ \Rightarrow \ \left\{\begin{array}{l} \frac{x}{(y,y)} \in \mathring{f}(\max, \min) \\ f^s(x) \supseteq f^s(y) \\ \tilde{f}(x) \trianglerighteq \tilde{f}(y) \end{array}\right.\right). \tag{20}$$

$$(\forall x, y, z \in X)\left(x*y \le z \ \Rightarrow \ \left\{\begin{array}{l} \frac{x}{(y,z)} \in \mathring{f}(\max, \min) \\ f^s(x) \supseteq f^s(y) \cap f^s(z) \\ \tilde{f}(x) \trianglerighteq \mathrm{rmin}\{\tilde{f}(y), \tilde{f}(z)\} \end{array}\right.\right). \tag{21}$$

3 Dokdo implicative ideals

In this section, we define a Dokdo implicative ideal in a BCK-algebra, and investigate related properties. The symbol $(X, *, 0)$ and (U, X) in this section represent a BCK-algebra and a BCK-Dokdo universe, respectively, unless otherwise specified.

Definition 3.1. *A Dokdo structure $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ in (U, X) is called a Dokdo implicative ideal of $(X, *, 0)$ if it satisfies* (18) *and*

$$(\forall x, y, z \in X)\left(\begin{array}{l} \frac{x}{((x*(y*x))*z,\, z)} \in \mathring{f}(\max, \min), \\ f^s(x) \supseteq f^s((x*(y*x))*z) \cap f^s(z), \\ \tilde{f}(x) \trianglerighteq \mathrm{rmin}\{\tilde{f}((x*(y*x))*z), \tilde{f}(z)\} \end{array}\right). \tag{22}$$

Example 3.2. *Consider a BCK-algebra $(X, *)_{\lambda_0}$ in which $X = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ and $*$ is given by the following Cayley table (see [12]):*

$*$	λ_0	λ_1	λ_2	λ_3	λ_4
λ_0	λ_0	λ_0	λ_0	λ_0	λ_0
λ_1	λ_1	λ_0	λ_1	λ_1	λ_1
λ_2	λ_2	λ_2	λ_0	λ_2	λ_2
λ_3	λ_3	λ_3	λ_3	λ_0	λ_3
λ_4	λ_4	λ_4	λ_4	λ_4	λ_0

Define a Dokdo structure $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ *in* $(U := \mathbb{N}, X)$ *as follows:*

$$Dok_f := (\mathring{f}, f^s, \tilde{f}) : X \to ([-1,0] \times [0,1]) \times 2^U \times [[0,1]],$$
$$y \mapsto \begin{cases} ((-0.67, 0.78), 2\mathbb{Z}, [0.6, 0.8]) & \text{if } y = \lambda_0, \\ ((-0.46, 0.72), 8\mathbb{N}, [0.4, 0.7]) & \text{if } y = \lambda_1, \\ ((-0.62, 0.36), 4\mathbb{N}, [0.3, 0.6]) & \text{if } y = \lambda_2, \\ ((-0.38, 0.63), 4\mathbb{N}, [0.5, 0.7]) & \text{if } y = \lambda_3, \\ ((-0.58, 0.57), 4\mathbb{Z}, [0.3, 0.4]) & \text{if } y = \lambda_4. \end{cases}$$

It is routine to verify that $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ *is a Dokdo implicative ideal of* $(X, *)_{\lambda_0}$.

Proposition 3.3. *Every Dokdo implicative ideal* $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ *of* $(X, *, 0)$ *satisfies:*

$$(\forall x, y \in X) \left(\begin{array}{l} \frac{x}{(x*(y*x),\, x*(y*x))} \in \mathring{f}(\max, \min), \\ f^s(x) \supseteq f^s(x*(y*x)), \\ \tilde{f}(x) \trianglerighteq \tilde{f}(x*(y*x)) \end{array} \right), \tag{23}$$

$$(\forall x, y \in X) \left(\begin{array}{l} \frac{x*(x*y)}{(y*(y*x),\, y*(y*x))} \in \mathring{f}(\max, \min), \\ f^s(x*(x*y)) \supseteq f^s(y*(y*x)), \\ \tilde{f}(x*(x*y)) \trianglerighteq \tilde{f}(y*(y*x)) \end{array} \right). \tag{24}$$

Proof. Let $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ be a Dokdo implicative ideal of $(X, *, 0)$. The facts below are obtained using (2) and (18) after selecting $z = 0$ from (22).

$$\mathring{f}^-(x) \le \max\{\mathring{f}^-((x*(y*x))*0), \mathring{f}^-(0)\} = \mathring{f}^-(x*(y*x)),$$
$$\mathring{f}^+(x) \ge \min\{\mathring{f}^+((x*(y*x))*0), \mathring{f}^+(0)\} = \mathring{f}^+(x*(y*x)),$$

that is, $\frac{x}{(x*(y*x),\, x*(y*x))} \in \mathring{f}(\max, \min)$, and

$$f^s(x) \supseteq f^s((x*(y*x))*0) \cap f^s(0) = f^s(x*(y*x)) \cap f^s(0) = f^s(x*(y*x)),$$
$$\tilde{f}(x) \trianglerighteq \operatorname{rmin}\{\tilde{f}((x*(y*x))*0), \tilde{f}(0)\} = \operatorname{rmin}\{\tilde{f}(x*(y*x)), \tilde{f}(0)\} = \tilde{f}(x*(y*x)),$$

for all $x, y \in X$. This shows that (23) is valid. Now, the combination of (I1), (I2), (3) and (4) leads to

$$\begin{aligned} &(x*(x*y))*(y*(x*(x*y))) \le (x*(x*y))*(y*x) \\ &= (x*(y*x))*(x*y) \le y*(y*x), \end{aligned}$$

for all $x, y \in X$. Since $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ is a Dokdo ideal of $(X, *, 0)$ (see Theorem 3.4), it follows from (2) and (20) that

$$\frac{((x*(x*y))*(y*(x*(x*y))))*0}{(y*(y*x),\, y*(y*x))} = \frac{(x*(x*y))*(y*(x*(x*y)))}{(y*(y*x),\, y*(y*x))} \in \mathring{f}(\max, \min),$$

$$f^s(((x*(x*y))*(y*(x*(x*y))))*0) = f^s((x*(x*y))*(y*(x*(x*y)))) \supseteq f^s(y*(y*x)),$$

and

$$\tilde{f}(((x*(x*y))*(y*(x*(x*y))))*0) = \tilde{f}((x*(x*y))*(y*(x*(x*y)))) \trianglerighteq \tilde{f}(y*(y*x)),$$

for all $x, y \in X$. Hence the following is derived from these, (18) and (22).

$$\begin{aligned} \mathring{f}^-(x*(x*y)) &\le \max\{\mathring{f}^-(((x*(x*y))*(y*(x*(x*y))))*0), \mathring{f}^-(0)\} \\ &\le \max\{\mathring{f}^-(y*(y*x)), \mathring{f}^-(0)\} = \mathring{f}^-(y*(y*x)), \end{aligned}$$

$$\begin{aligned} \mathring{f}^+(x*(x*y)) &\ge \min\{\mathring{f}^+(((x*(x*y))*(y*(x*(x*y))))*0), \mathring{f}^+(0)\} \\ &\ge \min\{\mathring{f}^+(y*(y*x)), \mathring{f}^+(0)\} = \mathring{f}^+(y*(y*x)), \end{aligned}$$

that is, $\frac{x*(x*y)}{(y*(y*x),\, y*(y*x))} \in \mathring{f}(\max,\min)$, and

$$\begin{aligned} f^s(x*(x*y)) &\supseteq f^s(((x*(x*y))*(y*(x*(x*y))))*0) \cap f^s(0) \\ &\supseteq f^s(y*(y*x)) \cap f^s(0) = f^s(y*(y*x)), \end{aligned}$$

$$\begin{aligned} \tilde{f}(x*(x*y)) &\trianglerighteq \mathrm{rmin}\{\tilde{f}(((x*(x*y))*(y*(x*(x*y))))*0), \tilde{f}(0)\} \\ &\trianglerighteq \mathrm{rmin}\{\tilde{f}(y*(y*x)), \tilde{f}(0)\} = \tilde{f}(y*(y*x)), \end{aligned}$$

for all $x, y \in X$. Hence (24) is valid. □

We discuss the relationship between Dokdo subalgebra, Dokdo ideal, and Dokdo implicative ideal.

Theorem 3.4. *Every Dokdo implicative ideal is a Dokdo ideal.*

Proof. Let $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ be a Dokdo implicative ideal of $(X, *, 0)$. If we take $x = y$ in (22) and use (I3) and (2), then

$$\begin{aligned} &\tfrac{x}{(x*z,\, z)} = \tfrac{x}{((x*(x*x))*z,\, z)} \in \mathring{f}(\max,\min), \\ &f^s(x) \supseteq f^s((x*(x*x))*z) \cap f^s(z) = f^s(x*z) \cap f^s(z), \\ &\tilde{f}(x) \trianglerighteq \mathrm{rmin}\{\tilde{f}((x*(x*x))*z), \tilde{f}(z)\} = \mathrm{rmin}\{\tilde{f}(x*z), \tilde{f}(z)\}, \end{aligned}$$

for all $x, z \in X$. Therefore $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ is a Dokdo ideal of $(X, *, 0)$. □

Corollary 3.5. *Every Dokdo implicative ideal is a Dokdo subalgebra.*

The converse of Theorem 3.4 and Corollary 3.5 is not true as seen in the following example.

Example 3.6. *(1) Consider a BCK-algebra $(X,*)_{\lambda_0}$ in which $X = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ and $*$ is given by the following Cayley table (see [12]):*

$*$	λ_0	λ_1	λ_2	λ_3	λ_4
λ_0	λ_0	λ_0	λ_0	λ_0	λ_0
λ_1	λ_1	λ_0	λ_1	λ_0	λ_1
λ_2	λ_2	λ_2	λ_0	λ_2	λ_0
λ_3	λ_3	λ_1	λ_3	λ_0	λ_3
λ_4	λ_4	λ_4	λ_4	λ_4	λ_0

Define a Dokdo structure $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ in $(U := \mathbb{R}, X)$ as follows:

$$Dok_f := (\mathring{f}, f^s, \tilde{f}) : X \to ([-1,0] \times [0,1)) \times 2^U \times [[0,1]],$$
$$y \mapsto \begin{cases} ((-0.66, 0.78), 2\mathbb{R}, [0.62, 0.83]) & \text{if } y = \lambda_0, \\ ((-0.53, 0.52), 2\mathbb{N}, [0.44, 0.67]) & \text{if } y = \lambda_1, \\ ((-0.48, 0.71), 2\mathbb{Z}, [0.57, 0.79]) & \text{if } y = \lambda_2, \\ ((-0.53, 0.52), 2\mathbb{N}, [0.44, 0.67]) & \text{if } y = \lambda_3, \\ ((-0.39, 0.69), 4\mathbb{N}, [0.53, 0.74]) & \text{if } y = \lambda_4. \end{cases}$$

It is routine to verify that $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ is a Dokdo ideal of $(X, *, 0)$. But it is not a Dokdo implicative ideal of $(X, *, 0)$ because of $\frac{\lambda_1}{((\lambda_1*(\lambda_3*\lambda_1))*\lambda_2,\, \lambda_2)} = \frac{\lambda_4}{(\lambda_0,\, \lambda_2)} \notin \mathring{f}(\max,\min)$ and/or

$$\begin{aligned} \tilde{f}(\lambda_1) = [0.44, 0.67] &\ntrianglerighteq [0.57, 0.79] = \mathrm{rmin}\{\tilde{f}(\lambda_0), \tilde{f}(\lambda_2)\} \\ &= \mathrm{rmin}\{\tilde{f}((\lambda_1*(\lambda_3*\lambda_1))*\lambda_2), \tilde{f}(\lambda_2)\}. \end{aligned}$$

(2) Consider a BCK-algebra $(X,*)_{\lambda_0}$ in which $X = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3\}$ and $*$ is given by the following Cayley table (see [12]):

$*$	λ_0	λ_1	λ_2	λ_3
λ_0	λ_0	λ_0	λ_0	λ_0
λ_1	λ_1	λ_0	λ_0	λ_0
λ_2	λ_2	λ_1	λ_0	λ_1
λ_3	λ_3	λ_3	λ_3	λ_0

Define a Dokdo structure $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ *in* $(U := \mathbb{R}, X)$ *as follows:*

$$Dok_f := (\mathring{f}, f^s, \tilde{f}) : X \to ([-1,0] \times [0,1]) \times 2^U \times [[0,1]],$$
$$z \mapsto \begin{cases} ((-0.68, 0.79), \mathbb{R}, [0.64, 0.83]) & \text{if } z = \lambda_0, \\ ((-0.54, 0.62), \mathbb{N}, [0.34, 0.57]) & \text{if } z = \lambda_1, \\ ((-0.38, 0.41), \mathbb{Z}, [0.47, 0.69]) & \text{if } z = \lambda_2, \\ ((-0.43, 0.52), 2\mathbb{N}, [0.55, 0.77]) & \text{if } z = \lambda_3. \end{cases}$$

It is easy to check that $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ *is a Dokdo subalgebra of* $(X, *, 0)$*. But it is not a Dokdo ideal of* $(X, *, 0)$ *since* $f^s(\lambda_1) = \mathbb{N} \not\supseteq \mathbb{Z} = f^s(\lambda_1 * \lambda_2) \cap f^s(\lambda_2)$*. Also,* $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ *is not a Dokdo implicative ideal of* $(X, *, 0)$ *because of*

$$\tilde{f}(\lambda_1) = [0.34, 0.57] \not\trianglerighteq [0.55, 0.77] = \mathrm{rmin}\{\tilde{f}((\lambda_1 * (\lambda_2 * \lambda_1)) * \lambda_3), \tilde{f}(\lambda_3)\},$$

and/or $\frac{\lambda_2}{(\lambda_2 * (\lambda_1 * \lambda_2)) * \lambda_3,\ \lambda_3)} = \frac{\lambda_2}{(\lambda_1,\ \lambda_3)} \notin \mathring{f}(\max, \min)$.

Lemma 3.7. *If a Dokdo ideal* $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ *of* $(X, *, 0)$ *satisfies the condition* (23)*, then it satisfies:*

$$(\forall x, y \in X) \left(\begin{array}{l} \mathring{f}^-(x) = \mathring{f}^-(x * (y * x)),\ \mathring{f}^+(x) = \mathring{f}^+(x * (y * x)) \\ f^s(x) = f^s(x * (y * x)),\ \tilde{f}(x) = \tilde{f}(x * (y * x)) \end{array} \right). \tag{25}$$

Proof. Assume that a Dokdo ideal $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ of $(X, *, 0)$ satisfies the condition (23). Since $x * (y * x) \leq x$ for all $x, y \in X$, it follows from (20) that $\frac{x*(y*x)}{(x,x)} \in \mathring{f}(\max, \min)$, $f^s(x * (y * x)) \supseteq f^s(x)$ and $\tilde{f}(x * (y * x)) \trianglerighteq \tilde{f}(x)$. Combining this with (23) yields the result (25). □

Proposition 3.8. *Every Dokdo implicative ideal* $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ *of* $(X, *, 0)$ *satisfies:*

$$(\forall x, y, z, u \in X) \left((x * (y * x)) * z \leq u \ \Rightarrow \begin{cases} \frac{x}{(z,\, u)} \in \mathring{f}(\max, \min), \\ f^s(x) \supseteq f^s(z) \cap f^s(u), \\ \tilde{f}(x) \trianglerighteq \mathrm{rmin}\{\tilde{f}(z), \tilde{f}(u)\} \end{cases} \right). \tag{26}$$

Proof. Let $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ be a Dokdo implicative ideal of $(X, *, 0)$. Then it satisfies the condition (23) (see Proposition 3.3) and is a Dokdo ideal of $(X, *, 0)$ (see Theorem 3.4). Let $x, y, z, u \in X$ be such that $(x * (y * x)) * z \leq u$. Then

$$\mathring{f}^-(x) = \mathring{f}^-(x * (y * x)) \leq \max\{\mathring{f}^-(z), \mathring{f}^-(u)\},$$
$$\mathring{f}^+(x) = \mathring{f}^+(x * (y * x)) \geq \min\{\mathring{f}^+(z), \mathring{f}^+(u)\},$$

that is, $\frac{x}{(z,\, u)} \in \mathring{f}(\max, \min)$, and $f^s(x) = f^s(x * (y * x)) \supseteq f^s(z) \cap f^s(u)$ and $\tilde{f}(x) = \tilde{f}(x * (y * x)) \trianglerighteq \mathrm{rmin}\{\tilde{f}(z), \tilde{f}(u)\}$ by (21) and Lemma 3.7. Therefore (26) is valid. □

We discuss conditions that allow Dokdo ideal and Dokdo subalgebra to be Dokdo implicative ideal.

Theorem 3.9. *If a Dokdo subalgebra* $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ *of* $(X, *, 0)$ *satisfies the condition* (26)*, then it is a Dokdo implicative ideal of* $(X, *, 0)$*.*

Proof. Let $Dok_f := (\mathring{f},\, f^s,\, \tilde{f})$ be a Dokdo subalgebra of $(X,*,0)$ that satisfies (26). Combinations of (I3), (15), (16), and (17) derive $\frac{0}{(x,\,x)} = \frac{x*x}{(x,\,x)} \in \mathring{f}(\max,\min)$, $f^s(0) = f^s(x*x) \supseteq f^s(x) \cap f^s(x) = f^s(x)$ and $\tilde{f}(0) = \tilde{f}(x*x) \trianglerighteq \operatorname{rmin}\{\tilde{f}(x), \tilde{f}(x)\} = \tilde{f}(x)$ for all $x \in X$. Since $(x*(y*x))*((x*(y*x))*z) \leq z$ for all $x,y,z \in X$, it follows from (26) that

$$\begin{aligned}&\tfrac{x}{((x*(y*x))*z,\,z)} \in \mathring{f}(\max,\min),\\ &f^s(x) \supseteq f^s((x*(y*x))*z) \cap f^s(z),\\ &\tilde{f}(x) \trianglerighteq \operatorname{rmin}\{\tilde{f}((x*(y*x))*z), \tilde{f}(z)\}\end{aligned}$$

for all $x,y,z \in X$. Therefore $Dok_f := (\mathring{f},\, f^s,\, \tilde{f})$ is a Dokdo implicative ideal of $(X,*,0)$. □

Theorem 3.10. *If a Dokdo ideal $Dok_f := (\mathring{f},\, f^s,\, \tilde{f})$ of $(X,*,0)$ satisfies the condition (23), then it is a Dokdo implicative ideal of $(X,*,0)$.*

Proof. Assume that a Dokdo ideal $Dok_f := (\mathring{f},\, f^s,\, \tilde{f})$ of $(X,*,0)$ satisfies the condition (23). Using (23) and (19), we have

$$\begin{aligned}&\mathring{f}^-(x) \leq \mathring{f}^-(x*(y*x)) \leq \max\{\mathring{f}^-((x*(y*x))*z), \mathring{f}^-(z)\},\\ &\mathring{f}^+(x) \geq \mathring{f}^+(x*(y*x)) \geq \min\{\mathring{f}^+((x*(y*x))*z), \mathring{f}^+(z)\},\end{aligned}$$

that is, $\frac{x}{((x*(y*x))*z,\,z)} \in \mathring{f}(\max,\min)$, and

$$f^s(x) \supseteq f^s(x*(y*x)) \supseteq f^s((x*(y*x))*z) \cap f^s(z)$$

and $\tilde{f}(x) \trianglerighteq \tilde{f}(x*(y*x)) \trianglerighteq \operatorname{rmin}\{\tilde{f}((x*(y*x))*z), \tilde{f}(z)\}$ for all $x,y,z \in X$. Hence $Dok_f := (\mathring{f},\, f^s,\, \tilde{f})$ is a Dokdo implicative ideal of $(X,*,0)$. □

Theorem 3.11. *In an implicative BCK-algebra, every Dokdo ideal is a Dokdo implicative ideal.*

Proof. Let $Dok_f := (\mathring{f},\, f^s,\, \tilde{f})$ be a Dokdo ideal of an implicative BCK-algebra $(X,*,0)$. Using (I3), (4) and the implicativity of $(X,*,0)$, we get $x*((x*(y*x))*z) \leq z$ for all $x,y,z \in X$. It follows from (21) that $\frac{x}{((x*(y*x))*z,z)} \in \mathring{f}(\max,\min)$, $f^s(x) \supseteq f^s((x*(y*x))*z) \cap f^s(z)$ and $\tilde{f}(x) \trianglerighteq \operatorname{rmin}\{\tilde{f}((x*(y*x))*z), \tilde{f}(z)\}$ for all $x,y,z \in X$. Therefore $Dok_f := (\mathring{f},\, f^s,\, \tilde{f})$ is a Dokdo implicative ideal of $(X,*,0)$. □

Corollary 3.12. *If $(X,*,0)$ satisfies one of the following three conditions:*

1. *the zero ideal $\{0\}$ is implicative,*
2. *every ideal of $(X,*,0)$ is implicative,*
3. *for every $a \in X$, the set $\{x \in X \mid x \leq a\}$ is an implicative ideal of $(X,*,0)$,*

then every Dokdo ideal is a Dokdo implicative ideal.

Lemma 3.13 ([12])**.** *An ideal A of $(X,*,0)$ is implicative if and only if it satisfies:*

$$(\forall x,y \in X)(x*(y*x) \in A \;\Rightarrow\; x \in A). \tag{27}$$

Theorem 3.14. *If $Dok_f := (\mathring{f},\, f^s,\, \tilde{f})$ is a Dokdo implicative ideal of $(X,*,0)$, then the nonempty sets $\mathring{f}(t^-,t^+)$, f^s_α and $\tilde{f}_{\tilde{a}}$ are implicative ideals of $(X,*,0)$ for all $(t^-,t^+) \in [-1,0]\times[0,1]$, $\alpha \in 2^U$ and $\tilde{a} = [a_l,a_r]$.*

Proof. Assume that $Dok_f := (\mathring{f},\, f^s,\, \tilde{f})$ is a Dokdo implicative ideal of $(X,*,0)$. Then it is a Dokdo ideal of $(X,*,0)$ (see Theorem 3.4). Let $(t^-,t^+) \in [-1,0]\times[0,1]$, $\alpha \in 2^U$ and $\tilde{a} = [a_l,a_r]$ be such that $\mathring{f}(t^-,t^+)$, f^s_α and $\tilde{f}_{\tilde{a}}$ are nonempty. It is clear that $0 \in \mathring{f}(t^-,t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$ by (18). Let $x,y \in X$ be such that

$y \in \mathring{f}(t^-,t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$ and $x*y \in \mathring{f}(t^-,t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$. Then $\mathring{f}^-(y) \leq t^-$, $\mathring{f}^+(y) \geq t^+$, $f^s(y) \supseteq \alpha$, $\tilde{f}(y) \trianglerighteq \tilde{a}$, $\mathring{f}^-(x*y) \leq t^-$, $\mathring{f}^+(x*y) \geq t^+$, $f^s(x*y) \supseteq \alpha$, and $\tilde{f}(x*y) \trianglerighteq \tilde{a}$. It follows from (19) that

$$\mathring{f}^-(x) \leq \max\{\mathring{f}^-(x*y), \mathring{f}^-(y)\} \leq t^-,\ \mathring{f}^+(x) \geq \min\{\mathring{f}^+(x*y), \mathring{f}^+(y)\} \geq t^+,$$
$$f^s(x) \supseteq f^s(x*y) \cap f^s(y) \supseteq \alpha,\ \tilde{f}(x) \trianglerighteq \operatorname{rmin}\{\tilde{f}(x*y), \tilde{f}(y)\} \trianglerighteq \tilde{a}.$$

Hence $x \in \mathring{f}(t^-,t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$, and thus $\mathring{f}(t^-,t^+)$, f^s_α and $\tilde{f}_{\tilde{a}}$ are ideals of $(X,*,0)$. Let $x,y \in X$ be such that $x*(y*x) \in \mathring{f}(t^-,t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$. Then $\mathring{f}^-(x*(y*x)) \leq t^-$, $\mathring{f}^+(x*(y*x)) \geq t^+$, $f^s(x*(y*x)) \supseteq \alpha$ and $\tilde{f}(x*(y*x)) \trianglerighteq \tilde{a}$. It follows from Proposition 3.3 that

$$\mathring{f}^-(x) \leq \mathring{f}^-(x*(y*x)) \leq t^-,\ \mathring{f}^+(x) \geq \mathring{f}^+(x*(y*x)) \geq t^+,$$
$$f^s(x) \supseteq f^s(x*(y*x)) \supseteq \alpha,\ \tilde{f}(x) \trianglerighteq \tilde{f}(x*(y*x)) \trianglerighteq \tilde{a}$$

which shows that $x \in \mathring{f}(t^-,t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$. Therefore $\mathring{f}(t^-,t^+)$, f^s_α and $\tilde{f}_{\tilde{a}}$ are implicative ideals of $(X,*,0)$ by Lemma 3.13. □

The converse of Theorem 3.14 may not be true as seen in the following example.

Example 3.15. *Consider a BCK-algebra $(X,*)_{\lambda_0}$ in which $X = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ and $*$ is given by the following Cayley table (see [12]):*

$*$	λ_0	λ_1	λ_2	λ_3	λ_4
λ_0	λ_0	λ_0	λ_0	λ_0	λ_0
λ_1	λ_1	λ_0	λ_1	λ_0	λ_1
λ_2	λ_2	λ_2	λ_0	λ_0	λ_2
λ_3	λ_3	λ_2	λ_1	λ_0	λ_3
λ_4	λ_4	λ_4	λ_4	λ_4	λ_0

Define a Dokdo structure $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ in $(U := \mathbb{Z}, X)$ as follows:

$$Dok_f := (\mathring{f}, f^s, \tilde{f}) : X \to ([-1,0] \times [0,1]) \times 2^U \times [[0,1]],$$
$$y \mapsto \begin{cases} ((-0.27, 0.75), 2\mathbb{Z}, [0.56, 0.78]) & \text{if } y = \lambda_0, \\ ((-0.36, 0.42), 8\mathbb{Z}, [0.33, 0.47]) & \text{if } y = \lambda_1, \\ ((-0.63, 0.42), 4\mathbb{Z}, [0.41, 0.56]) & \text{if } y = \lambda_2, \\ ((-0.63, 0.67), 8\mathbb{Z}, [0.33, 0.47]) & \text{if } y = \lambda_3, \\ ((-0.48, 0.67), 8\mathbb{N}, [0.49, 0.64]) & \text{if } y = \lambda_4. \end{cases}$$

It is routine to check that the nonempty sets $\mathring{f}(t^-,t^+)$, f^s_α and $\tilde{f}_{\tilde{a}}$ are implicative ideals of $(X,*)_{\lambda_0}$ for all $(t^-,t^+) \in [-1,0] \times [0,1]$, $\alpha \in 2^U$ and $\tilde{a} = [a_l, a_r]$. But $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ is not a Dokdo implicative ideal of $(X,*)_{\lambda_0}$ because of

$$\frac{\lambda_1}{(((\lambda_1*\lambda_2)*\lambda_1)*\lambda_3,\ \lambda_3)} = \frac{\lambda_1}{(\lambda_0, \lambda_3)} \notin \mathring{f}(\max, \min).$$

It provides a condition under which the converse of Theorem 3.14 may be true in the theorem below.

Theorem 3.16. *Given a Dokdo structure $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ in (U, X), if the nonempty sets $\mathring{f}(t^-)$, $\mathring{f}(t^+)$, f^s_α and $\tilde{f}_{\tilde{a}}$ are implicative ideals of $(X,*,0)$ for all $(t^-,t^+) \in [-1,0] \times [0,1]$, $\alpha \in 2^U$ and $\tilde{a} = [a_l, a_r]$, then $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ is a Dokdo implicative ideal of $(X,*,0)$.*

Proof. Assume that $\mathring{f}(t^-)$, $\mathring{f}(t^+)$, f^s_α and $\tilde{f}_{\tilde{a}}$ are nonempty implicative ideals of $(X,*,0)$ for all $(t^-,t^+) \in [-1,0] \times [0,1]$, $\alpha \in 2^U$ and $\tilde{a} = [a_l, a_r]$. Then they are ideals and so subalgebras of $(X,*,0)$. Let $x,y \in X$ be such that $Dok_f(x) := (\mathring{f}(x),\ f^s(x),\ \tilde{f}(x)) = ((t^-_x, t^+_x), \alpha_x, \tilde{a}_x)$ and $Dok_f(y) := (\mathring{f}(y),\ f^s(y),\ \tilde{f}(y)) = ((t^-_y, t^+_y), \alpha_y, \tilde{a}_y)$. If we take

$$((t^-,t^+), \alpha, \tilde{a}) = ((\max\{t^-_x, t^-_y\}, \min\{t^+_x, t^+_y\}), \alpha_x \cap \alpha_y, \operatorname{rmin}\{\tilde{a}_x, \tilde{a}_y\}),$$

then $x, y \in \mathring{f}(t^-) \cap \mathring{f}(t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$ and so $x * y \in \mathring{f}(t^-) \cap \mathring{f}(t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$. If we put $x = y$ and use (I3), then $\frac{0}{(x,x)} \in \mathring{f}(\max, \min)$, $f^s(0) \supseteq f^s(x)$ and $\tilde{f}(0) \trianglerighteq \tilde{f}(x)$ for all $x \in X$. Let $x, y, z \in X$ be such that $Dok_f(c) := (\mathring{f}(c),\ f^s(c),\ \tilde{f}(c)) = ((t^-_c, t^+_c), \alpha_c, \tilde{a}_c)$ and $Dok_f(z) := (\mathring{f}(z),\ f^s(z),\ \tilde{f}(z)) = ((t^-_z, t^+_z), \alpha_z, \tilde{a}_z)$ where $c := (x * (y * x)) * z$. Taking

$$((t^-, t^+), \alpha, \tilde{a}) = ((\max\{t^-_c, t^-_z\}, \min\{t^+_c, t^+_z\}), \alpha_c \cap \alpha_z, \mathrm{rmin}\{\tilde{a}_c, \tilde{a}_z\})$$

implies that $c := (x * (y * x)) * z \in \mathring{f}(t^-) \cap \mathring{f}(t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$ and $z \in \mathring{f}(t^-) \cap \mathring{f}(t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$. It follows that $x \in \mathring{f}(t^-) \cap \mathring{f}(t^+) \cap f^s_\alpha \cap \tilde{f}_{\tilde{a}}$. Hence

$$\begin{aligned} &\tfrac{x}{((x*(y*x))*z,\, z)} \in \mathring{f}(\max, \min), \\ &f^s(x) \supseteq f^s((x * (y * x)) * z) \cap f^s(z), \\ &\tilde{f}(x) \trianglerighteq \mathrm{rmin}\{\tilde{f}((x * (y * x)) * z), \tilde{f}(z)\}. \end{aligned}$$

Therefore $Dok_f := (\mathring{f},\ f^s,\ \tilde{f})$ is a Dokdo implicative ideal of $(X, *, 0)$. □

4 Conclusions

With the aim of helping solve the problem of potential uncertainty in everyday life, Jun introduced a new hybrid structure called Dokdo structure, where "Dokdo" is the name of the most beautiful island in Korea, and applied it to BCK/BCI-algebras. In this manuscript, we used the Dokdo structure to study implicative ideals in BCK-algebras. We introduced the concept of Dokdo implicative ideals in BCK-algebras and investigated the relevant properties. We discussed the relationship between Dokdo subalgebra, Dokdo ideal, and Dokdo implicative ideal. We provided conditions that allow Dokdo ideal and Dokdo subalgebra to be Dokdo implicative ideal.

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