Journal of Algebraic Hyperstructures and Logical Algebras

Volume 3, Number 3, (2022), pp. 23-32

A combinatorial property of logical algebras

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Abstract

Logic gives a technique for the articial intelligence to make the computers simulate human being in dealing with certainty and uncertainty in information. Various logical algebras have been proposed and researched as the semantical systems of non-classical logical systems. In this paper using the concept of commutator, we introduce the Engel algebra and then study a condition on infinite subsets of infinite algebras. We also show that some logical algebras satisfy to this condition but do not have the properties associated with that condition.

Article Information

Corresponding Author:
A. Mosayebi Dorcheh;
Received: December 2021;
Revised: February 2022;
Accepted: March 2022;
Paper type: Original.

Keywords:

Commutator, Engel graph, infinite subsets, nilpotent, invariant relation.

1 Introduction

BCI-algebras as a class of logical algebras are the algebraic formulations of the set difference together with its properties in set theory and the implicational functor in logical systems. Their names are originated from the combinators B, C, K and I in combinatory logic.

BCK-algebras and BCI-algebras are abbreviated to two B-algebras. The former was raised in 1966 by Y. Imai and K. Iseki, Japanese mathematicians, and the latter was put forward in the same year due to K. Iseki.

In set theory, there are three most elementary and fundamental operations. They are the union, intersection and set difference. If we consider those three operations, then we have the notion of Boolean algebras. If we take both of the union and intersection, then as a general algebra, the notion of lattices is obtained. Moreover, if we consider the union or the intersection alone, we have the notion of upper semilattices or lower semilattices. However, the set difference together with its properties had not been considered systematically before K. Iseki. There are some systems

https://doi.org/10.52547/HATEF.JAHLA.3.3.2

which contain the only implicational functor among logical functors. Hyper logical algebras were first studied in 2000 by Borzooei et al. They applied the concept of hyperstructures to one of the logical algebraic structures known as the BCK-algebra, and introduced two generalizations of them called the hyper BCK-algebra and hyper K-algebra.

They also introduced the notions of hyper I-algebras and hyper K-algebras and the union of two hyper K-algebras. Then they stated and proved some related theorems. In particular, by some examples they shew that these definitions are different from the notion of hyper BCK-algebras, however any hyper BCK-algebra is a hyper K-algebra. Then by defining the concept of hyper K-algebra product of two hyper K-algebras, they gave a theorem which shows that the relation between the hyper K-ideal of the given hyper K-algebras and the hyper K-ideals of their product [2, 3].

Now, in this paper we introduce the autocommutator using the automorphisms. Then we study some conditions on infinite subsets of infinite BCI-algebras. We also using the concept of commutator, introduce the Engel algebra and then study a graph related to an algebra.

2 Preliminaries

In this section, we recall some definitions which will need in the next sections.

A BCI-algebra is an algebraic structure $(A, *, 0)$ of type $(2, 0)$ such that, for all $x, y, z \in A$:

(BCI-1) $((x * y) * (x * z)) * (z * y) = 0$,

(BCI-2) $x * 0 = x$,

(BCI-3) $x * y = 0 = y * x$ imply $x = y$.

Define a binary relation $\leq$ on A by which $x \leq y$ if and only if $x * y = 0$ for any $x, y \in A$. Then $(A, \leq)$ is a partially ordered set with 0 as a minimal element in the meaning that $x \leq 0$ implies $x = 0$ for any $x \in A$. Given an element x in a BCI-algebra A, if it satisfies $0 * x = 0$ (that is, $x \geq 0$), the element x is call a positive element of A. A non-vacuous subset X of a BCI-algebra A is a subalgebra of A if and only if X is closed under the $*$ on A. If A is a BCI-algebra, then B and P are subalgebras of A, where B is the set of all positive elements of A, and P the set of all minimal elements of A.

Let $(A, *, 0)$ be a BCI-algebra. Then A is called a BCK-algebra if $0 * x = 0$, for any $x \in A$. An algebra $(A, *, 0)$ of type $(2, 0)$ is a P-algebra if and only if it satisfies the following conditions, for any $x, y, z \in A$,

(1) $(x * y) * (x * z) = z * y$,

(2) $x * 0 = x$.

A BCK-algebra A is called commutative if $x * (x * y) = y * (y * x)$, for any $x, y \in A$.
A BCI-algebra A is P-algebra if and only if $0 * (0 * x) = x$, for all $x \in A$.

Let A be a BCI-algebra and for any $x, y \in A$, define, $x * y^0 = x$, $x * y^{n+1} = (x * y^n) * y$, where $n \in \mathbb{N}$, if there is a natural number k such that $0 * x^k = 0$, the element x is called a nilpotent element of A, and the least natural number satisfying $0 * x^k = 0$ is called the period of x.

A BCI-algebra A is called nilpotent if every element in A is nilpotent.
A BCK-algebra A is called positive imlicative if $x * y = x * y^2$, for any $x, y \in A$.

Proposition 2.1. [4] *Let $(A_1, *_1, 0_1)$ and $(A_2, *_2, 0_2)$ be BCI-algebras, and let A denote the Cartesian product $A_1 \times A_2$ of A_1 and A_2, i.e, $A = \{(x, y) \mid x \in A_1, y \in A_2\}$. Define a binary operation $*$ on A by $(x_1, y_1) * (x_2, y_2) = (x_1 *_1 x_2, y_1 *_2 y_2)$. Then, $(A, *, 0)$ is a BCI-algebra, where $0 = (0_1, 0_2)$.*

Proposition 2.2. [4] *Let $(A_1, *_1, 0)$ and $(A_2, *_2, 0)$ be BCI-algebras such that $A_1 \cap A_2 = \{0\}$, define a binary operation $*$ on $A = A_1 \cup A_2$ by*

$$x * y = \begin{cases} x *_1 y & x, y \in A_1, \\ x *_2 y & x, y \in A_2, \\ 0 *_2 y & x \in A_1, y \in A_2 - \{0\}, \\ x & x \in A_2, y \in A_1. \end{cases}$$

Then, $(A, *, 0)$ is a BCI-algebra.

Definition 2.3. [4] *Let $(A_1, *_1, 0_1)$ and $(A_2, *_2, 0_2)$ be BCI-algebras. A mapping f from A_1 to A_2 is called a BCI-homomorphism if $f(x *_1 y) = f(x) *_2 f(y)$, for all $x, y \in A_1$. An isomorphism means that it is both of surjective and injective. If A is a BCI-algebra, an automorphism of A is an isomorphism from A to A. The set of automorphisms of A denoted by $\mathrm{Aut}(A)$.*
Let A be a non-empty set and $*$ a map from $A \times A$ to $P(A) - \{\emptyset\}$, where $P(A)$ denotes the power set of A. For two subsets X and Y of A, denote by $X * Y$ the set $\bigcup_{x \in X, y \in Y} x * y$.

A hyper P-algebra (HP-algebra) is a non-empty set A endowed with a hyperoperation $*$ and a constant 0 such that, for all $x, y, z \in A$, it satisfying in the following conditions:

(1) $(x * y) * (x * z) = z * y$,

(2) $x * 0 = \{x\}$.

Theorem 2.4. *Let A be an HP-algebra and let $x, y, z \in A$, then*

(i) $0 * 0 = \{0\}, x * x = \{0\}, 0 * (0 * x) = \{x\}, x * (x * y) = \{y\}$,

(ii) *if* $0 \in x * y$, *then* $x = y$,

(iii) *if* $x * y = x * z$, *then* $y = z$.

Proof. This follows from the definition. □

3 A graph associated with a BCI-algebra

In this section, using the concept of commutator, we introduce the Engel elements of a BCI-algebra and then we introduce a graph related to algebras. But before that we need some properties about commutators and we prove them.

Definition 3.1. *Let $(A, *, 0)$ be a BCI-algebra and let $x_1, x_2, \ldots, x_n, x, y$ be elements of A, we define the commutator x and y as follows:*

$$[x, y] = (x * (0 * y)) * (y * (0 * x)).$$

More generally, we define inductively $[x_1, \ldots, x_n]$ as follows:

$$[x_1, \ldots, x_n] = [[x_1, \ldots, x_{n-1}], x_n].$$

A useful shorthand notation is $[x,_n y] = [x, \underbrace{y, \ldots, y}_{n}]$.

Example 3.2. *The set $A = \{0, 1, 2, 3\}$ with the operation $*$ given by Table 1,*

Table 1

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	2	0	0
3	3	2	1	0

froms a BCI-algebra (See [4]). Then $[2, 1] = 2$ and $[2,_n 1] = 2$, for all $n \in \mathbb{N}$. Also $[2, 3] = 0$, $[0, x] = 0$, for all $x \in A$.

Lemma 3.3. *Let $(A, *, 0)$ be a BCI-algebra. Then*

(i) *$B = \{[x, 0] \mid x \in A\}$, where B is the set of all positive elements of A,*

(ii) *$0 * [x, y] = [0 * x, 0 * y]$, for any $x, y \in A$,*

(iii) *$[0, x] = [x, x] = [0 * x, x] = 0$, for any $x \in A$,*

(iv) *If $x, y \in B$, then $[x, y] \in B$,*

(v) *$A = P$ if and only if $[x, y] = 0$, for any $x, y \in A$, where P is the set of all minimal elements of A.*

Proof. (i) We have $0 * [x, 0] = 0$ and if $b \in B$, then there exists $x \in A$ such that $b = [x, 0]$.

(iv) By (ii) we have $0 * [x, y] = 0$ and so $[x, y] \in B$.

□

Proposition 3.4. *Let A be a BCK-algebra. Then,*

(i) *A is commutative if and only if for any $x, y \in A$, $[x, [x, y]] = [y, [y, x]]$.*

(ii) *The following conditions are equivalent:*

(1) *A is positive implicative,*

(2) *$[x, y] = [x,_2 y]$, for all $x, y \in A$,*

(3) *$[x, y, z] = [[x, z], [y, z]]$, for all $x, y, z \in A$.*

Proof. A is commutative if and only if for any $x, y \in A$, $[x, [x, y]] = x * (x * y) = y * (y * x) = [y, [y, x]]$. □

As an application of the above definition, we give the Engel graph, (for Engel graph associated with a group, see [1]).

Definition 3.5. *Let A be a BCI-algebra, then an element x of A is called left Engel if for every element $a \in A$, there exists a positive integer n such that $[a,_n x] = 0$. If the integer n is fixed for any element a, then the element x is called left n-Engel. The set of all left Engel (n-Engel) elements of A is denoted by $L(A)$ ($L_n(A)$). A BCI-algebra A is called an Engel (n-Engel) BCI-algebra, if $L(A) = A - \{0\}$ ($L_n(A) = A - \{0\}$). Associate with a non-Engel A a graph $G(A)$ as follows:*
Take $V = A - (L(A) \cup \{0\})$ as vertices of $G(A)$ and join two distinct vertices x and y whenever $[x,_n y] \neq 0 \neq [y,_n x]$, for all positive integers n. We call $G(A)$, the Engel graph of A.

Example 3.6. *Let $A = \{0, 1, 2\}$. Define an operation $*$ on A by Table 2.*

Table 2

$*$	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

Then $(A, *, 0)$ is a BCK-algebra (see [4]*). It is easy to verify that $L(A) = \emptyset$ and thus $V = \{1, 2\}$ and Engel graph $G(A)$ is shown in Figure 1.*

Figure 1: Engel graph of A

Example 3.7. *Let $A = \{0, a, b, c\}$ in which $*$ is given by Table 3.*

Table 3

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	b
c	c	c	c	0

Simple calculations show that $(A, *, 0)$ is a BCK-algebra and $V = A - (L(A) \cup \{0\}) = \{a, b, c\}$. We obtain the Engel graph in Figure 2.

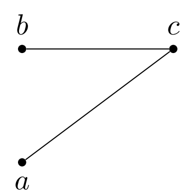

Figure 2

Example 3.8. *Let $A = \{0, a_1, a_2, a_3, a_4, a_5, a_6\}$. Define an operation $*$ on A by*

$$x * y = \begin{cases} 0 & x = y, \\ x & x \neq y. \end{cases}$$

Then $(A, *, 0)$ is a BCK-algebra. It is not difficult to verify that $V = \{a_1, a_2, a_3, a_4, a_5, a_6\}$. Thus the Engel graph $G(A)$ is represented by the Figure 3.

Theorem 3.9. *Let A be a positive implicative BCK-algebra, and $|A| \geq 3$ and let $G(A) = (V, E)$ be the Engel graph of A. Then*

(*i*) $|V| = |A| - 1$ *or* $|V| = |A| - 2$,

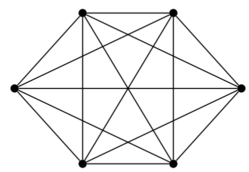

Figure 3

(*ii*) *Let x, y be distinct elements of V, then $xy \in E$ if and only if x and y are not comparable.*

Proof. (*i*) Let a be the greatest element, if it exists, then a is comparable with all elements of the poset A and so $L(A) = \{a\}$, this completes the proof of part (i).

(*ii*) Let x, y be any two comparable points, therefore $0 = [x, y] = [x,_n y]$ or $0 = [y, x] = [y,_n x]$, for all $n \in \mathbb{N}$. Thus comparable elements are not joined in the Engel graph. Hence the result follows.

□

Example 3.10. *The set $A = \{0, 1, 2, 3, 4\}$ together with the operation $*$ on A given by Figure 4*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	2	0
3	3	3	3	0	0
4	4	4	3	2	0

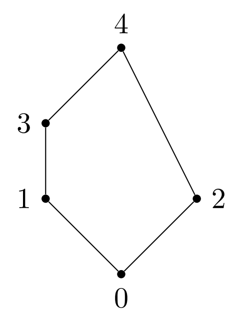

Figure 4

forms a positive implicative BCK-algebra, (See [4]). By routine verification, the element 4 is the greatest element of A. But this implies that, $L(A) = \{4\}$ and $V = \{1, 2, 3\}$, as shown in the diagram below:

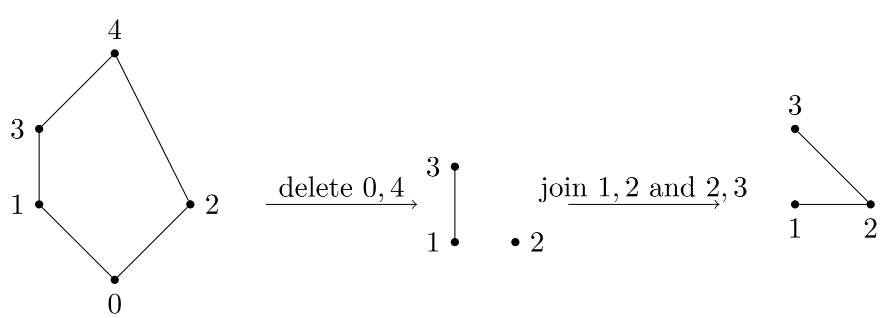

Figure 5

Definition 3.11. *Let $(A, *, 0)$ be a BCI-algebra and let $x \in A$, $\alpha \in \mathrm{Aut}(A)$. We define $[x, \alpha] = x * \alpha(x)$ and will call an autocommutator of x and α. Inductively, for all $\alpha_1, \alpha_2, \ldots, \alpha_n \in \mathrm{Aut}(A)$,*

$$[x, \alpha_1, \alpha_2, \ldots, \alpha_n] = [[x, \alpha_1, \alpha_2, \ldots, \alpha_{n-1}], \alpha_n].$$

If $\alpha_1 = \alpha_2 = \cdots = \alpha_n$*, then we denote* $[x, \alpha_1, \alpha_2, \ldots, \alpha_n]$ *by* $[x,_n \alpha_1]$*. A is called auto-Engel if* $[x,_n \alpha] = 0$*, for any* x, α*.*

Example 3.12. *Let A be the BCI-algebra as in Example 3.7. Then simple calculations show that* $[x,_n \alpha] = 0$*, for any* $x \in A$*,* $\alpha \in \mathrm{Aut}(A)$*, and thus A is auto-Engel which is not Engel.*

Proposition 3.13. *Suppose that* $(A, *, 0)$ *is a BCI-algebra,* $x, y \in A$ *and* $n \in \mathbb{N}$*, and let* $\alpha \in \mathrm{Aut}(A)$*. Then*

(*i*) $[0, \alpha] = [0,_n \alpha] = [x,_n I] = 0$*, where I is the identity of the group* $\mathrm{Aut}(A)$*,*

(*ii*) $\alpha([x,_n \alpha]) = [\alpha(x),_n \alpha]$*,* $0 * \alpha(x)^n = \alpha(0 * x^n)$*,*

(*iii*) *If* $x \in B$*, then* $[x, \alpha] \in B$*,*

(*iv*) *If* $[\alpha, x] = \alpha(x) * x$*, then* $[\alpha, x] = [\alpha(x), \alpha^{-1}] = \alpha([x, \alpha^{-1}])$*,*

(*v*) $\alpha([x, y]) = [\alpha(x), \alpha(y)]$*.*

Proof. (ii) By the definition, we obtain $\alpha([x, \alpha]) = [\alpha(x), \alpha]$. Therefore

$$\alpha([x,_n \ \alpha]) = [\alpha([x,_{n-1} \ \alpha]), \alpha] = \cdots = [\alpha(x),_n \ \alpha],$$

for $n > 1$. Also by the definition, we have

$$\alpha(0 * x^n) = \alpha((0 * x^{n-1}) * x) = \cdots = (0 * \alpha(x)^{n-1}) * \alpha(x) = 0 * \alpha(x)^n.$$

(iii) If $x \in B$, then $[x, \alpha] \geq 0$.

□

Definition 3.14. *Let A be a BCI-algebra and let R be a relation on A. Then R is called an invariant relation, if for all* $\alpha \in \mathrm{Aut}(A)$*, we have* $\alpha(R) \subseteq R$*, where*

$$\alpha(R) = \{\alpha(x, y) = (\alpha(x), \alpha(y)) \mid (x, y) \in R\}.$$

A nonempty subset K of A is called characteristic in A if for all $\alpha \in \mathrm{Aut}(A)$*,* $\alpha(K) \subseteq K$*.*

Example 3.15. *Let A be a BCI-algebra. Define a binary relation R on A as follows:* $(x, y) \in R$ *if and only if* $x * y \in B$ *and* $y * x \in B$ *for all* $x, y \in A$*. Then R is an invariant relation. Also B is characteristic.*

Example 3.16. *Let A be the algebra as in Example 3.6. Then* $\mathrm{Aut}(A) = \{I, (1\ 2)\}$ *and set* $\{1, 2\}$ *is characteristic but* $\{0, 1\}$ *is not characteristic.*

Example 3.17. *Suppose that K is a characteristic subalgebra of a BCI-algebra A. Then, the relation* $R = \{(x, y) \in A \times A \mid x * y, y * x \in K\}$ *is an invariant relation.*

Conversely, if R be an invariant equivalence relation on a BCI-algebra A, then the set

$$K = \{x * y \mid (x, y) \in R\},$$

is a characteristic subset of A.

Proposition 3.18. *Let* K_1 *and* K_2 *be two nonempty subsets of BCI-algebra* A_1 *and* A_2*, respectively, and let* $K_1 \times K_2$ *be a characteristic subset of* $A_1 \times A_2$*. Then* K_i *is characteristic.*

Proof. Let $\alpha_i \in \mathrm{Aut}(A_i)$, $i = 1, 2$. we define $\alpha = (\alpha_1, \alpha_2)$ by $\alpha((x, y)) = (\alpha_1(x), \alpha_2(y))$, clearly $\alpha \in \mathrm{Aut}(A_1 \times A_2)$. Now, we obtain

$$\alpha_1(K_1) \cong \alpha(K_1 \times \{0\}) \subseteq K_1 \times \{0\} \cong K_1,$$

and

$$\alpha_2(K_2) \cong \alpha(\{0\} \times K_2) \subseteq \{0\} \times K_2 \cong K_2,$$

as required. □

Example 3.19. *Let $A = \{0, a\}$ in which $*$ is given by the Table 4.*

Table 4

$*$	0	a
0	0	a
a	a	0

Then $(A, *, 0)$ is a BCI-algebra. Also $\{0\}$ and $\{a\}$ are characteristic subsets of A but $\{(0, a)\} = \{0\} \times \{a\}$ is not characteristic.

The above example, shows that the converse of the above proposition, is not necessarily true.

4 A condition on infinite subsets

In this section, we study algebras with a condition on infinite subsets of them.

Theorem 4.1. *Let A be an infinite P-algebra and let $n \in \mathbb{N}$ and let for any infinite subset X of A there exists $x \in X$ such that $0 * x^n = 0$, then A is nilpotent of class n.*

Proof. Consider the set $N = \{x \in A \mid 0 * x^n = 0\}$. If $A - N$ is infinite, then by the hypothesis there is element $a \in A - N$ such that $0 * a^n = 0$, a contradiction. Thus $A - N$ is finite. Now, suppose, for a contradiction, that $a \in A - N$ and $b \in N$, then $0 * (a * (0 * b))^n = 0 * a^n \neq 0$ and hence $a * (0 * b) \in A - N$, on the other hand if $a * (0 * b_1) = a * (0 * b_2)$, for $b_1, b_2 \in N$, then $b_1 = b_2$ and so $D = \{a * (0 * b) \mid b \in N\}$ is an infinite subset of $A - N$, a contradiction. Therefore $A = N$. □

The next example points out that a BCI-algebra with condition of infinite subset need not be nilpotent.

Example 4.2. *Let $A = \{0, 1, 2, 3, \ldots\}$. Define an operation $*$ on A by*

$$x * y = \begin{cases} 0 & x \leq y, y \neq 1 \neq x \text{ or } x = y = 1, \\ x & x > y, y \neq 1, \\ 1 & x \neq 1, y = 1 \text{ or } x = 1, y \neq 1. \end{cases}$$

It is easy to verify that the algebra A is a BCI-algebra which for any infinite subset X of A there exists $x \in X$, such that $0 * x^n = 0$, but obviously, 1 is not nilpotent and therefore A is not nilpotent.

Theorem 4.3. *Let A be an infinite P-algebra and let for any infinite subset X of A there exists $x \in X$ such that $0 * x = x$, then for all $x \in A$, $0 * x = x$.*

Proof. Define the set $S = \{x \in A \mid 0 * x = x\}$. By property of infinite subset, one can see that S is an infinite subset of A. Let $t \in A \setminus S$ and consider the infinite set $T = \{s * t \mid s \in S\}$. Therefore by the property A, there exists $s \in S$ such that $0 * (s * t) = s * t$. But $0 * (s * t) = s * (0 * t)$ and so $0 = (s * (0 * t)) * (s * t) = t * (0 * t)$. Thus $t \in S \cap (A - S)$, a contradiction. Therefore $A = S$. □

Question 4.4. *In which class of algebras does the infinite subsets condition result in the corresponding property?*

Example 4.5. *Let A be the algebra as in Example 4.2. Then for every two infinite subsets X and Y of A there exist $x \in X$, $y \in Y$ such that $[x, y] = 0$, but $[2, 0] = 2$.*

Theorem 4.6. *Let A_1 and A_2 be infinite BCI-algebras and let $A_1 \times A_2$ satisfies the infinite subsets condition. Then A_i is too, $i = 1, 2$.*

Theorem 4.7. *Let A be an infinite P-algebra, $n \in \mathbb{N}$, and for every two infinite subsets X and Y of A there exist $x \in X$, $y \in Y$ such that $0 * x^n = 0 * y^n$. Then for all $x, y \in A - \{0\}$, $0 * x^n = 0 * y^n$.*

Proof. Put $U = \{\{x, y\} \in A^{(2)} \mid 0 * x^n = 0 * y^n\}$ and $V = A^{(2)} - U$, where $A^{(2)}$ is the set of all 2-element subsets of $A - \{0\}$. Since V is finite, so U is infinite, and suppose, for a contradiction, that $\{a, b\} \in V$. Let T be an infinite subset of A, such that $T^{(2)} \subseteq U$, and X and Y be infinite distinct subsets of T. Now, consider the infinite sets $X_1 = \{x * (0 * a) \mid x \in X\}$ and $Y_1 = \{y * (0 * b) \mid y \in Y\}$ and using the property A, we find the elements $x \in X$ and $y \in Y$ such that

$$0 * (x * (0 * a))^n = 0 * (y * (0 * b))^n.$$

But $\{x, y\} \in U$ and thus $0 * a^n = 0 * b^n$. So $\{a, b\} \in U$, a contradiction. □

Theorem 4.8. *Let A be an infinite P-algebra and $\alpha \in \mathrm{Aut}(A)$, and let for every infinite subset X of A there exists $x \in X$ such that $[x, \alpha] = 0$, then for all $x \in A$, $[x, \alpha] = 0$.*

Proof. The proof is similar to the proof of Theorem 4.3. □

Theorem 4.9. *Let A be an infinite P-algebra and let for any infinite subset X of A there exists $x \in X$ such that $[x, \alpha] = 0$ for all $\alpha \in \mathrm{Aut}(A)$. Then $\mathrm{Aut}(A) = \{I\}$.*

Proof. Let $\alpha \in Aut(A)$, then by Theorem 4.8, we have $\alpha = I$. □

Theorem 4.10. *Let A be an infinite P-algebra, and let B be an infinite subalgebra of A. If for any infinite subset X of B there exists $x \in X$ such that $\alpha(x) \in X$, for all $\alpha \in \mathrm{Aut}(A)$, then B is characteristic in A.*

Proof. Consider the set $C = \{x \in B \mid \alpha(x) \in B, \forall \alpha \in \mathrm{Aut}(A)\}$. By property of infinite subset, C is an infinite subset of B. Indeed, suppose on the contrary there exists $t \in B - C$ if

$$T = \{t * (0 * a) \mid a \in C\},$$

therefore T is an infinite subset of $B - C$ and by the property B there exists $a \in C$ such that $\alpha(t*(0*a)) \in B$, for all $\alpha \in \mathrm{Aut}(A)$ and so $\alpha(t) = (\alpha(t)*(0*\alpha(a)))*\alpha(a) \in B$, for all $\alpha \in \mathrm{Aut}(A)$, a contradiction. It follows that $B = C$. □

Corollary 4.11. *Let A be an infinite P-algebra and let for any infinite subset X of A there exists $x \in X$ such that $\alpha(x) \in X$, for all $\alpha \in \mathrm{Aut}(A)$, then every infinite subalgebra of A is a characteristic subalgebra of A.*

5 Conclusion

In this article, some concepts such as commutator, Engel graph and autocommutator are introduced and examples of their aplications are stated. The following items have been obtained from them:

(i) A graph related to a non-Engel algebra is introduced.

(ii) Engel algebras have been defined and studied.

(iii) The condition of infinite subsets is expressed on infinite algebras and their properties are studied.

References

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