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Irregular vague graphs

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Abstract

A vague graph is a generalized structure of a fuzzy graph that gives more precision, flexibility and compatibility to a system when compared with systems that are designed using fuzzy graphs. In this paper, the new concepts of (totally) irregular, strongly (totally) irregular, highly totally irregular, neighborly totally irregular, edge-irregular of vague graphs are introduced and generalized, and some properties and related results are investigated. Then, we present the concepts of homomorphism, weak isomorphism, co-weak isomorphism and isomorphism of irregular vague graphs and some results on (total) domination number, (total) 2-domination number, (total) cobondage number and (total) 2-cobondage number. Finally, one of their applications related to location map of Fire Stations and Emergency Medical centers in urban regions of a metropolis is presented.

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1 Introduction

Euler first introduced the concept of graph theory in 1736. The theory of graph is regarded as an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operations research, optimization and computer science. Gau and Buehrer [9] proposed the concept of the vague set by replacing the value of an element in a set with a subinterval of $[0, 1]$. Namely, a true-membership function $t_v(x)$ and a false membership function $f_v(x)$ are used to describe the boundaries of the membership degree. Accordingly, Ramakrishna [11] introduced the concept of vague graphs, along with some of their properties. Akram, Feng, Sawar and Jun [1] introduced the concept of highly and neighborly irregular of vague graphs. Borzooei and Rashmanlou [5] studied different types of dominating set in vague graphs. Borzooei and Banitalebi [3] introduced concepts of additions of an arc, cobondage sets, and cobondage numbers in vague graphs. Borzooei and Banitalebi [4] introduced concepts 2-dominating sets, 2-domination

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numbers, 2-cobondage sets and 2-cobondage numbers in vague graphs. The concepts of dominating sets, cobondage sets are considered as the fundemental concepts in the theory of vague graphs and have applications in several fields, especially in the fields of operations research, neural networks, electrical networks and monitoring communication.
According to the definition of the concept of regular vague graph in [8], it is possible to define the class of irregular vague graphs, which is much wider than the class of regular vague graph, using the concepts of vertex and edge degree.
The purpose of this paper is to discuss concepts (totally) irregular, strongly (totally) irregular, highly totally irregular, neighborly totally irregular, edge-irregular, homomorphism, weak isomorphism, co-weak isomorphism and isomorphism of vague graphs. Finally, we will provide a model for optimizing the domination parameters while maintaining stability in the cobondage parameters using these concepts.

2 Preliminaries

A *fuzzy graph* [10, 12] $G = (\sigma, \mu)$ on simple graph $G^* = (V, E)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that, for any $uv \in E$, $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$, where $\wedge$ denote minimum. A *vague set* A in an ordinary finite non-empty set X, is a pair (t_A, f_A), where $t_A : X \to [0, 1]$ and $f_A : X \to [0, 1]$ are true and false membership functions, respectively, such that for all $x \in X$, $0 \leq t_A(x) + f_A(x) \leq 1$. Note that $t_A(x)$ is considered as the lower bound for positive degree of membership of x in A and $f_A(x)$ is the lower bound for negative degree of membership of x in A. So, the degree of membership of x in the vague set A, is characterized by the interval $[t_A(x), 1 - f_A(x)]$.
A *vague graph* [11] on simple graph $G^* = (V, E)$ is defined to be a pair $G = (A, B)$, where $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on E such that for any edge $xy \in E$,

$$t_B(xy) \leq \min\{t_A(x), t_A(y)\}, \quad f_B(xy) \geq \max\{f_A(x), f_A(y)\}.$$

The underlying *crisp graph* of a vague graph $G = (A, B)$, is the graph $H = (V_1, E_1)$, where $V_1 = \{v \in V : t_A(v) > 0 \textit{ and } f_A(v) > 0\} \subseteq V$ and $E_1 = \big\{uv \in E : \ t_B(uv) > 0, \ f_B(uv) > 0\big\} \subseteq E$. A vague graph G is called *complete* [11] if for any $v_i, v_j \in V$,

$$t_B(v_iv_j) = \min\{t_A(v_i), t_A(v_j)\}, \quad f_B(v_iv_j) = \max\{f_A(v_i), f_A(v_j)\}.$$

Definition 2.1. [5] *Let $G = (A, B)$ be a vague graph on simple graph $G^* = (V, E)$. Then:*
(i) The vertex cardinality of G is defined by,

$$|V| = \sum_{v_i \in V} \big(\frac{t_A(v_i) + (1 - f_A(v_i))}{2}\big).$$

(ii) The edge cardinality of G is defined by,

$$|E| = \sum_{v_iv_j \in E} \big(\frac{t_B(v_iv_j) + (1 - f_B(v_iv_j))}{2}\big).$$

(iii) The cardinality of G is defined by,

$$|G| = |V| + |E|.$$

(iv) For any $U \subseteq V$, the vertex cardinality of U is denoted by $O(U)$ and defined by,

$$O(U) = \sum_{v_i \in U} \Big(\frac{t_A(v_i) + (1 - f_A(v_i))}{2}\Big).$$

(v) For any $F \subseteq E$, the edge cardinality of F is denoted by $S(F)$ and defined by,

$$S(F) = \sum_{v_i v_j \in F} \Big(\frac{t_B(v_i v_j) + (1 - f_B(v_i v_j))}{2}\Big).$$

Definition 2.2. [3, 5] *Let $G = (A, B)$ be a vague graph on simple graph $G^* = (V, E)$. Then:*
(i) An edge $e = uv \in E$ is called a strong edge in G if $t_B(uv) \geq (t_B)^\infty(uv)$ and $f_B(uv) \leq (f_B)^\infty(uv)$, where

$$(t_B)^\infty(uv) = \max\{(t_B)^k(uv)|\ k = 1, 2, \cdots, n\}, \quad (f_B)^\infty(uv) = \min\{(f_B)^k(uv)|\ k = 1, 2, \cdots, n\},$$

and

$$t_B^k(uv) = \min\{t_B(ux_1), t_B(x_1x_2), \cdots, t_B(x_{k-1}v)\big|u, x_1, \cdots, x_{k-1}, v \in V, k = 1, 2, \cdots, n\},$$

$$f_B^k(uv) = \max\{f_B(ux_1), f_B(x_1x_2), \cdots, f_B(x_{k-1}v)\big|u, x_1, \cdots, x_{k-1}, v \in V, k = 1, 2, \cdots, n\}.$$

(ii) The neighborhood of $u \in V$ is denoted by $N(u)$ and defined as follows:

$$N(u) = \{v \in V \mid uv \text{ is a strong edge in } G\}.$$

(iii) We say that u dominate v in G if there exists a strong edge between u and v.
(iv) $S \subset V$ is called a dominating set in G if for any $v \in V \setminus S$, there exists $u \in S$ such that u dominates v.
(v) A dominating set S in G is called a minimal dominating set if no proper subset of S is a dominating set.
(vi) The lower domination number of G is denoted by $d_V(G)$ and defined by,

$$d_V(G) = \min\{O(D)\big|D \text{ is a minimal domination set of } G\}.$$

(vii) The upper domination number of G is denoted by $D_V(G)$ and defined by,

$$D_V(G) = \max\{O(D)\big|D \text{ is a minimal domination set of } G\}.$$

(viii) The domination number of G is denoted by $\Delta_V(G)$ and defined by,

$$\Delta_V(G) = \frac{d_V(G) + D_V(G)}{2}.$$

Definition 2.3. [3, 5] *Let $G = (A, B)$ be a vague graph on simple graph $G^* = (V, E)$ without isolated vertices. Then:*
(i) $S \subset V$ is called a total dominating set in G if for any $v \in V$, there exists $u \in S$ such that $u \neq v$ and u dominates v.
(ii) A total dominating set S of G is called a minimal total dominating set if no proper subset of

S is a total dominating set of G.
(iii) Minimum vertex cardinality among all minimal total dominating sets of G is called lower total domination number of G and is denoted by $t_V(G)$.
(iv) Maximum vertex cardinality among all minimal total dominating sets of G is called upper total domination number of G and is denoted by $T_V(G)$.
(v) The t-domination number of G is denoted by $\Delta^t_V(G)$ and defined as follows,

$$\Delta^t_V(G) = \frac{t_V(G) + T_V(G)}{2}.$$

Definition 2.4. [3] *Let $G = (A, B)$ be a vague graph on simple graph $G^* = (V, E)$. Then:*
(i) The cobondage set of a vague graph G is the set C of additional strong edges of G_e, that reduces the domination number of G. i.e.

$$\Delta_V(G_C) < \Delta_V(G).$$

(ii) A cobondage set C of G is said to be a minimal cobondage set if no proper subset of X is a cobondage set.
(iii) The lower cobondage number of G is denoted by $b_E(G)$ and defined by,

$$b_E(G) = \min\{S(C)\big|C \text{ is a minimal cobondage set of } G\}.$$

(iv) The upper cobondage number of G is denoted by $B_E(G)$ and defined by,

$$B_E(G) = \max\{S(C)\big|C \text{ is a minimal cobondage set of } G\}.$$

(v) The t-cobondage set of a vague graph G is the set C_t of additional strong arcs to G that reduces the t-domination number, i.e, $\Delta^t_V(G_{C_t}) < \Delta^t_V(G)$.
(vi) A t-cobondage set C_t of G is called a minimal t-cobondage set if no proper subset of C_t is a t-cobondage set.
(vii) Minimum edge cardinality among all minimal t-cobondage sets of G is called a lower t-cobondage number of G and is denoted by $b^t_E(G)$.
$(viii)$ Maximum edge cardinality among all minimal t-cobondage sets of G is called an upper t-cobondage number of G and is denoted by $B^t_E(G)$.

Definition 2.5. [3, 5] *Let $G = (A, B)$ be a vague graph on simple graph $G^* = (V, E)$. Then:*
(i) Two vertices $u,\ v \in V$ are called independent if there is not any strong edge between them.
(ii) $S \subset V$ is called an independent set in G if for any $u, v \in S$, $t_B(uv) < (t_B)^\infty(uv)$ and $f_B(uv) > (f_B)^\infty(uv)$.
(iii) An independent set S in G is called a maximal independent set if for any vertex $v \in V \setminus S$, the set $S \cup \{v\}$ is not independent.
(iv) Minimum vertex cardinality among all maximal independent sets is called a lower independent number of G and is denoted by $i_V(G)$.
(v) Maximum vertex cardinality among all maximal independent sets is called an upper independent number of G and is denoted by $I_V(G)$.
(vi) The independent number of G is denoted by $I(G)$ and defined as follows,

$$I(G) = \frac{i_V(G) + I_V(G)}{2}.$$

Definition 2.6. [6, 8] *Let $G = (A, B)$ be a vague graph on simple graph $G^* = (V, E)$. Then:*
(i) The t-degree of a vertex u is denoted by $d_t(u)$ and defined by,

$$d_t(u) = \sum_{v \in N(u)} t_B(uv).$$

(ii) The f-degree of a vertex u is denoted by $d_f(u)$ and defined by,

$$d_f(u) = \sum_{v \in N(u)} f_B(uv).$$

(iii) The degree of a vertex u is denoted by $d(u)$ and defined by,

$$d(u) = [d_t(u), d_f(u)].$$

(iv) The total degree of a vertex u is denoted by $td(u)$ and defined by,

$$td(u) = [d_t(u) + t_A(u), d_f(u) + f_A(u)].$$

(v) The t-degree of an edge $e_{ij} \in E$ is denoted by $d_t(e_{ij})$ and defined by,

$$d_t(e_{ij}) = d_t(v_i) + d_t(v_j) - 2t_B(v_i v_j).$$

(vi) The f-degree of an edge $e_{ij} \in E$ is denoted by $d_f(e_{ij})$ and defined by,

$$d_f(e_{ij}) = d_f(v_i) + d_f(v_j) - 2f_B(v_i v_j).$$

(vii) The degree of an edge $e_{ij} \in E$ is denoted by $d(e_{ij})$ and defined by,

$$d(e_{ij}) = [d_t(e_{ij}), d_f(e_{ij})].$$

Definition 2.7. [1, 2, 7] *Let $G(A, B)$ be a connected vague graph on $G^*(A, B)$. Then:*
(i) G is said to be a highly irregular vague graph if every vertex of G is adjacent to vertices with distinct degrees.
(ii) G is said to be a neighborly irregular vague graph if every pair of adjacent vertices of G have distinct degrees.

Notation. From now on, in this paper, we let $G = (A, B)$ be a connected vague graph on a simple graph $G^* = (V, E)$.

3 New concepts of irregularity in vague graphs

In this section, we discuss about the new concepts of irregular vague graphs.

Definition 3.1. *Let G be a connected vague graph. Then:*
(i) G is called an irregular vague graph if there exists a vertex of G which is adjacent to vertices with distinct degrees.
(ii) G is called a strongly irregular vague graph if every pair of vertices of G have distinct degrees.
(iii) G is called a totally irregular vague graph if there exists a vertex of G which is adjacent to vertices with distinct total degrees.
(iv) G is called a highly totally irregular vague graph if every vertex of G is adjacent to vertices

with distinct total degrees.
(v) G is called a strongly totally irregular vague graph if every pair of vertices of G have distinct total degrees.
(vi) G is called a neighborly totally irregular vague graph if every pair of adjacent vertices of G have distinct totally degrees.
(vii) G is called an edge-irregular vague graph if there exists an edge of G which is adjacent to edges with distinct degrees.

Example 3.2. *Consider a vague graph G in Figure 1. Obviously,*

$$d_t(a) = 0.3,\ d_t(b) = 0.4,\ d_t(c) = 0.2,\ d_t(d) = 0.3,\ d_t(e) = 0.2.$$

$$d_f(a) = 0.7,\ d_f(b) = 1.3,\ d_f(c) = 1.1,\ d_f(d) = 1,\ d_f(e) = 0.5.$$

Then,

$$d(a) = [0.3, 0.7],\ d(b) = [0.4, 1.3],\ d(c) = [0.2, 1.1],\ d(d) = [0.3, 1],\ d(e) = [0.2, 0.5],$$

and

$$td_t(a) = 0.6,\ td_t(b) = 0.8,\ td_t(c) = 0.3,\ td_t(d) = 0.5,\ td_t(e) = 0.5.$$

$$td_f(a) = 1.4,\ td_f(b) = 1.9,\ td_f(c) = 1.4,\ td_f(d) = 1.5,\ td_f(e) = 0.9.$$

Then,

$$td(a) = [0.6, 1.4],\ td(b) = [0.8, 1.9],\ td(c) = [0.3, 1.4],\ td(d) = [0.5, 1.5],\ td(e) = [0.5, 0.9].$$

Therefore, G is a strongly irregular and strongly totally irregular vague graph.

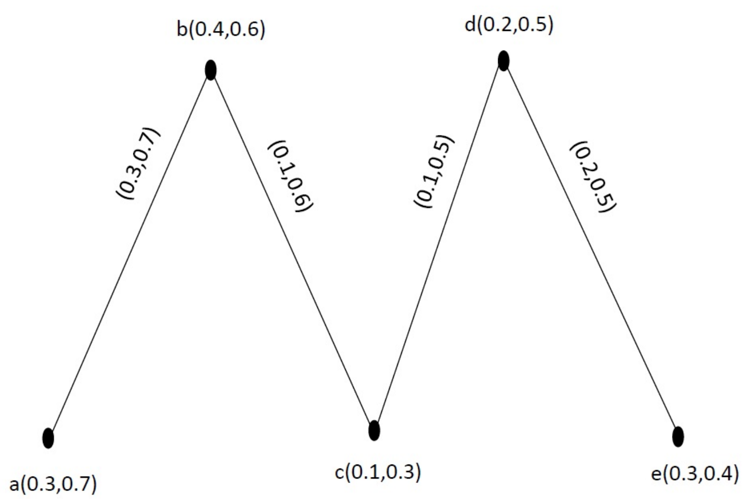

Figure 1: Vague graph G.

Example 3.3. *Consider a vague graph G in Figure 2. Obviously,*

$$d_t(a) = 0.3,\ d_t(b) = 0.5,\ d_t(c) = 0.7,\ d_t(d) = 0.5,\ d_t(e) = 0.8.$$

$$d_f(a) = 1.6,\ d_f(b) = 1.8,\ d_f(c) = 2,\ d_f(d) = 1.8,\ d_f(e) = 2.8.$$

Then,

$$d(a) = [0.3, 1.6],\ d(b) = [0.5, 1.8],\ d(c) = [0.7, 2],\ d(d) = [0.5, 1.8],\ d(e) = [0.8, 2.8],$$

and

$$td_t(a) = 0.4,\ td_t(b) = 0.7,\ td_t(c) = 1,\ td_t(d) = 0.7,\ td_t(e) = 1.1.$$
$$td_f(a) = 1.9,\ td_f(b) = 2.2,\ td_f(c) = 2.6,\ td_f(d) = 2.3,\ td_f(e) = 3.5.$$

Then,

$$td(a) = [0.4, 1.9],\ td(b) = [0.7, 2.2],\ td(c) = [1, 2.6],\ td(d) = [0.7, 2.3],\ td(e) = [1.1, 3.5].$$

Therefore, G is a highly irregular and strongly totally irregular vague graph.

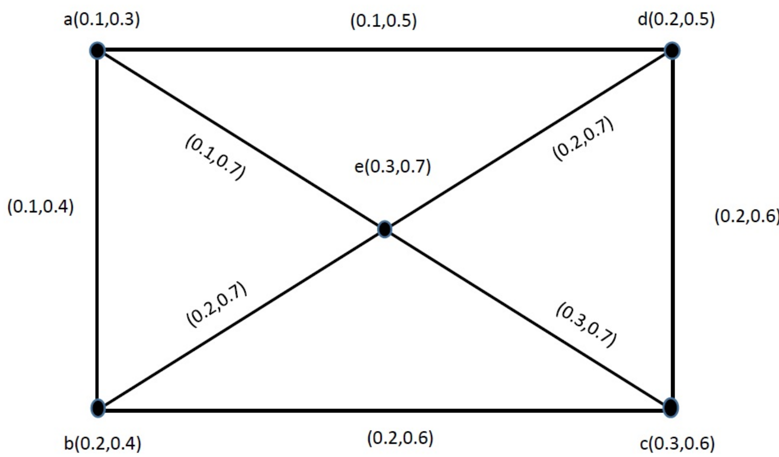

Figure 2: Vague graph G.

Example 3.4. *Consider a vague graph G in Figure 2. Obviously,*

$$d_t(ab) = 0.6,\ d_t(ae) = 0.9,\ d_t(ad) = 0.6,\ d_t(be) = 0.9,\ d_t(bc) = 0.8,\ d_t(ce) = 0.9,\ d_t(cd) = 0.8,\ d_t(de) = 0.9.$$
$$d_f(ab) = 2.6,\ d_f(ae) = 3,\ d_f(ad) = 2.4,\ d_f(be) = 3.2,\ d_f(bc) = 2.4,\ d_f(ce) = 3.4,\ d_f(cd) = 2.6,\ d_f(de) = 3.2.$$

Then,

$$d(ab) = [0.6, 2.6],\ d(ae) = [0.9, 3],\ d(ad) = [0.6, 2.4],\ d(be) = [0.9, 3.2],$$
$$d(bc) = [0.8, 2.4],\ d(ce) = [0.9, 3.4],\ d(cd) = [0.8, 2.6],\ d(de) = [0.9, 3.2],$$

and

$$td_t(ab) = 0.7,\ td_t(ae) = 1,\ td_t(ad) = 0.7,\ td_t(be) = 1.1,\ td_t(bc) = 1,\ td_t(ce) = 1.2,\ td_t(cd) = 1,\ td_t(de) = 1.1.$$
$$td_f(ab) = 3,\ td_f(ae) = 3.7,\ td_f(ad) = 2.9,\ td_f(be) = 3.9,\ td_f(bc) = 3.1,$$
$$td_f(ce) = 4.1,\ td_f(cd) = 3.2,\ td_f(de) = 3.9.$$

Then,

$$td(ab) = [0.7, 3],\ td(ae) = [1, 3.7],\ td(ad) = [0.7, 2.9],\ td(be) = [1.1, 3.9],\ td(bc) = [1, 3.1],$$
$$td(ce) = [1.2, 4.1],\ td(cd) = [1, 3.2],\ td(de) = [1.1, 3.9].$$

Therefore, G is an edge-irregular vague graph.

Remark 3.5. *The Example 3.3 shows that if G is a highly irregular vague graph, then G is not a necessarily strongly irregular vague graph.*

Theorem 3.6. *Let G be a complete vague graph and the set of all vertices of G be denoted by $V = \{v_1, v_2, v_3, ..., v_n\}$ such that $t_A(v_1) < t_A(v_2) < t_A(v_3) < ... < t_A(v_n)$. Then G is an irregular vague graph.*

Proof. Let G be a complete vague graph. Then for any $v_i, v_j \in V$,

$$f_B(v_iv_j) = f_A(v_i) \vee f_A(v_j), \quad t_B(v_iv_j) = t_A(v_i) \wedge t_A(v_j).$$

Also, for $v_1, v_3 \in V$, we have

$$d_t(v_3) = t_A(v_1) + t_A(v_2) + (n-3)t_A(v_3), \quad d_t(v_1) = (n-1)t_A(v_1).$$

Therefore, $d_t(v_1) \neq d_t(v_3)$ and v_2 adjacent to v_1 and v_3 with distinct degrees. □

Note. If f and t are two strictly increasing or strictly decreasing chains on vertex-set V of G, then Theorem 3.5 is true.

Theorem 3.7. *Let G be a connected vague graph and G^* be a cycle with $2n+1$ vertices. If alternate edges take same membership values, then G is an irregular vague graph.*

Proof. Assum that alternate edges take same membership values. Then,

$$t_B(e_i) = \begin{cases} \alpha_1, & \text{if i is odd,} \\ \alpha_2, & \text{if i is even,} \end{cases}$$

and

$$f_B(e_i) = \begin{cases} \beta_1, & \text{if i is odd,} \\ \beta_2, & \text{if i is even,} \end{cases}$$

and so,

$$d(v_1) = (2\alpha_1, 2\beta_1), \quad d(v_i) = (\alpha_1 + \alpha_2, \beta_1 + \beta_2),$$

where $i = 2, 3, ..., 2n+1$. Hence there exists a vertex v_2 which is adjacent to v_1, v_3 with distinct degrees. Therefore, G is an irregular vague graph. □

Theorem 3.8. *Let G be a connected vague graph and G^* be a cycle with $2n+1$ vertices. If alternate edges take same membership values, then G is an edge-irregular vague graph.*

Proof. Assum that alternate edges take same membership values. Then,

$$t_B(e_i) = \begin{cases} \alpha_1, & \text{if i is odd,} \\ \alpha_2, & \text{if i is even,} \end{cases}$$

and

$$f_B(e_i) = \begin{cases} \beta_1, & \text{if i is odd,} \\ \beta_2, & \text{if i is even,} \end{cases}$$

and so,

$$d_G(e_1) = (\alpha_1 + \alpha_2, \beta_1 + \beta_2), \quad d_G(e_i) = (2\alpha_1 + 2\alpha_2 - 2t_B(e_i), 2\beta_1 + 2\beta_2 - 2f_B(e_i)),$$

where $i = 2, 3, ..., 2n+1$. Hence there exists an edge e_2 which is adjacent to e_1, e_3 with distinct degrees. Therefore, G is an edge-irregular vague graph. □

Theorem 3.9. *Let G be a connected vague graph and $A = (t_A, f_A)$ be a constant function. Then the following expressions are equivalent:*

(i) G is a strongly irregular vague graph.

(ii) G is a strongly totally irregular vague graph.

Proof. Assume that for any $u \in V$,

$$A(u) = (t_A(u), f_A(u)) = (c_1, c_2),$$

where c_1 and c_2 are constant values. Then,

$$td_t(u) = d_t(u) + c_1, \quad td_f(u) = d_f(u) + c_2.$$

Hence for any $u, v \in V$, we have

$$td_t(u) \neq td_t(v) \Leftrightarrow d_t(u) \neq d_t(v),$$

and

$$td_f(u) \neq td_f(v) \Leftrightarrow d_f(u) \neq d_f(v).$$

□

Theorem 3.10. *Let G be a connected vague graph and $B = (t_B, f_B)$ be a constant function. Then the following expressions are equivalent:*

(i) G^* is a strongly irregular graph,

(ii) G is a strongly irregular vague graph.

Proof. Assume that for any $uv \in E$,

$$B(uv) = (t_B(uv), f_B(uv)) = (c_1, c_2),$$

where c_1 and c_2 are constant values. If G^* is a strongly irregular graph, then for any pair (u, v) of V, we have $d_{G^*}(u) \neq d_{G^*}(v)$. Otherwise $(t_B(uv), f_B(uv)) = (c_1, c_2)$, so that

$$d_t(u) = d_{G^*}(u).c_1, \quad d_f(u) = d_{G^*}(u).c_2.$$

Therefore,

$$d_{G^*}(u) \neq d_{G^*}(v) \Leftrightarrow d_t(u) \neq d_t(v) \Leftrightarrow d_f(u) \neq d_f(v).$$

□

Theorem 3.11. *Let G be a connected vague graph and $B = (t_B, f_B)$ be a constant function. If G is an edge-irregular vague graph, then G is an irregular vague graph.*

Proof. Assame that $B = (t_B, f_B)$ is a constant function. Suppose

$$B(e) = (t_B(e), f_B(e)) = (c_1, c_2),$$

for all $e \in E$, where c_1 and c_2 are constants. Then every edge of G is a strong edge. Now, by definition of edge degree, we have

$$d_t(e_{ij}) = (d_{G^*}(v_i) + d_{G^*}(v_j) - 2).c_1 \quad , \quad d_f(e_{ij}) = (d_{G^*}(v_i) + d_{G^*}(v_j) - 2).c_2.$$

Next by definition of vertex degree and from the above argument, the proof is clear. □

Note. If G is a complete vague graph, then concepts of strongly irregular and neighborly irregular are equivalent.

Proposition 3.12. *Let G be a neighborly irregular vague graph with $n \geq 4$ vertices and $B = (t_B, f_B)$ be a constant function. Then G^* is not a path and so there exists $v \in V$ such that $d_{G^*}(v) \geq 3$.*

Proof. The proof is straightforward. □

Definition 3.13. *A homomorphism h of irregular vague graphs G_1 and G_2 is a mapping $h : V_1 \to V_2$ which satisfies the following conditions:*
(i) $h(u) = u'$ where u adjacent to vertices with distinct degrees in G_1 and u' adjacent to vertices with distinct degrees in G_2,
(ii) for any $u \in V_1$, $t_{A_1}(u) \leq t_{A_2}(h(u))$, $f_{A_1}(u) \geq f_{A_2}(h(u))$,
(iii) for any $uv \in E_1$, $t_{B_1}(uv) \leq t_{B_2}(h(u)h(v))$, $f_{B_1}(uv) \geq f_{B_2}(h(u)h(v))$.

Definition 3.14. *Let h be a bijective homomorphism of irregular vague graphs G_1 and G_2. Then:*
(i) h is called a weak isomorphism of irregular vague graphs G_1 and G_2 if for any $u \in V_1$,

$$t_{A_1}(u) = t_{A_2}(h(u)),\ f_{A_1}(u) = f_{A_2}(h(u)),$$

(ii) h is called a co-weak isomorphism of irregular vague graphs G_1 and G_2 if for any $uv \in E_1$,

$$t_{B_1}(uv) = t_{B_2}(h(u)h(v)),\ f_{B_1}(uv) = f_{B_2}(h(u)h(v)),$$

(iii) h is called an isomorphism of irregular vague graphs G_1 and G_2 if it is a weak isomorphism and a co-weak isomorphism.

Notation. If h is an isomorphism of irregular vague graphs G_1 and G_2, then we say G_1 is an isomorphic irregular vague graph of G_2 and denote it by $G_1 \cong G_2$.

Example 3.15. *Consider irregular vague graphs G_1 and G_2 in Figure 3. If h is a mapping $h : V_1 \to V_2$ such that*

$$h(u_1) = a,\ h(u_2) = b,\ h(u_3) = c,\ h(u_4) = d,$$

then by routine computations, it is clear that h is a homomorphism of irregular vague graphs G_1 and G_2.

Example 3.16. *Consider irregular vague graphs G_1 and G_2 in Figure 4. If $h : V_1 \to V_2$ is a homomorphism of irregular vague graphs G_1 and G_2 such that*

$$h(u_1) = a,\ h(u_2) = e,\ h(u_3) = d,\ h(u_4) = c,\ h(u_5) = b,$$

then by routine computations, it is clear that h is a weak isomorphism of irregular vague graphs G_1 and G_2.

Example 3.17. *Consider irregular vague graphs G_1 and G_2 in Figure 5. If $h : V_1 \to V_2$ is a homomorphism of irregular vague graphs G_1 and G_2 such that*

$$h(u_1) = e,\ h(u_2) = d,\ h(u_3) = b,\ h(u_4) = c,\ h(u_5) = f,\ h(u_6) = a,$$

then by routine calculations, it is clear that h is a co-weak isomorphism of irregular vague graphs G_1 and G_2.

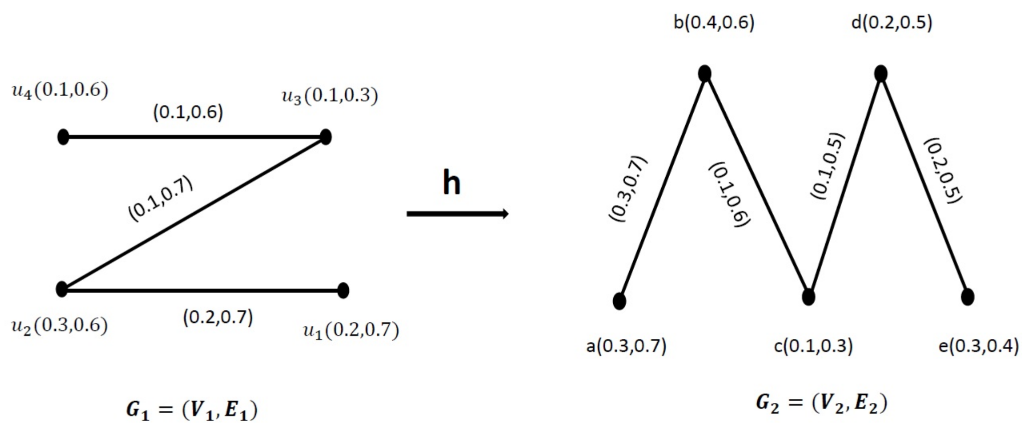

Figure 3: Irregular vague graphs G_1 and G_2.

Proposition 3.18. *If $G_1 \cong G_2$, then $O(G_1) = O(G_2)$ and $S(G_1) = S(G_2)$.*

Proof. Assum that h is an isomorphism of irregular vague graphs G_1 and G_2. Then for any $u, v \in V_1$,

$$t_{A_1}(u) = t_{A_2}(h(u)),\ f_{A_1}(u) = f_{A_2}(h(u)),$$

also,

$$t_{B_1}(uv) = t_{B_2}(h(u)h(v)),\ f_{B_1}(uv) = f_{B_2}(h(u)h(v)).$$

Hence,

$$O(G_1) = \big(\sum_{u\in V_1} t_{A_1}(u), \sum_{u\in V_1} f_{A_1}(u)\big) = \big(\sum_{u\in V_1} t_{A_2}(h(u)), \sum_{u\in V_1} f_{A_2}(h(u))\big) = O(G_2),$$

and

$$S(G_1) = \big(\sum_{uv\in E_1} t_{B_1}(uv), \sum_{uv\in E_1} f_{B_1}(uv)\big) = \big(\sum_{u,v\in V_1} t_{B_2}(h(u)h(v)), \sum_{u,v\in V_1} f_{B_2}(h(u)h(v))\big) = S(G_2).$$

□

Note. (i) If G_1 and G_2 are weak isomorphic, then $O(G_1) = O(G_2)$.
(ii) If G_1 and G_2 are co-weak isomorphic, then $S(G_1) = S(G_2)$.

Theorem 3.19. *If $G_1 \cong G_2$, then*
(i) $d_{V_1}(G_1) = d_{V_2}(G_2)$ and $D_{V_1}(G_1) = D_{V_2}(G_2)$, and so $\Delta_{V_1}(G_1) = \Delta_{V_2}(G_2)$.
(ii) $t_{V_1}(G_1) = t_{V_2}(G_2)$ and $T_{V_1}(G_1) = T_{V_2}(G_2)$, and so $\Delta^t_{V_1}(G_1) = \Delta^t_{V_2}(G_2)$.

Proof. (i) Assum that $h : V_1 \to V_2$ is an isomorphism of G_1 and G_2. If D_1 is a minimal dominating set of G_1, then by Definition 3.14, we get $h(D_1)$ is a minimal dominating set of G_2 and

$$\sum_{v_i\in D_1} \left(\frac{t_{A_1}(v_i) + (1 - f_{A_1}(v_i))}{2}\right) = \sum_{h(v_i)\in h(D_1)} \left(\frac{t_{A_2}(v_i) + (1 - f_{A_2}(v_i))}{2}\right).$$

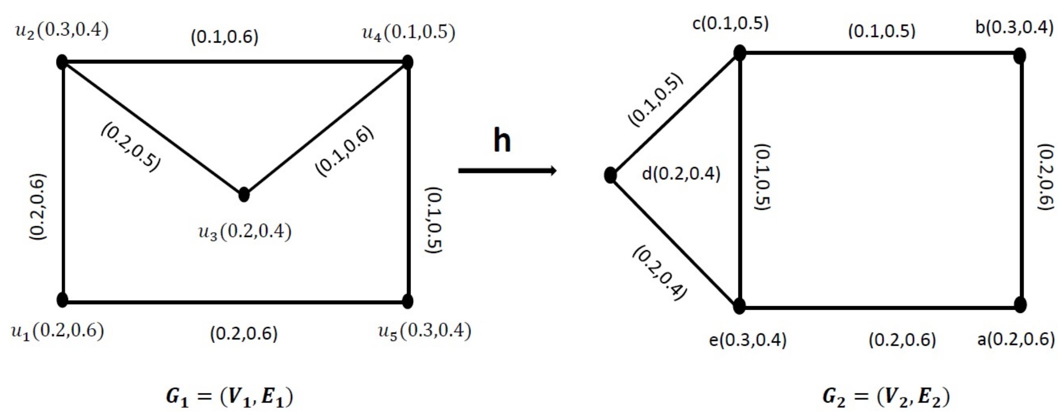

Figure 4: Irregular vague graphs G_1 and G_2.

Therefore, $d_{V_1}(G_1) = d_{V_2}(G_2)$, $D_{V_1}(G_1) = D_{V_2}(G_2)$ and so $\Delta_{V_1}(G_1) = \Delta_{V_2}(G_2)$.

(ii) Assum that $h : V_1 \to V_2$ is an isomorphism of G_1 and G_2. If D_1^t is a minimal total dominating set of G_1, then by Definition 3.14, we get $h(D_1^t)$ is a minimal total dominating set of G_2 and

$$\sum_{v_i \in D_1} \left(\frac{t_{A_1}(v_i) + (1 - f_{A_1}(v_i))}{2} \right) = \sum_{h(v_i) \in h(D_1^t)} \left(\frac{t_{A_2}(v_i) + (1 - f_{A_2}(v_i))}{2} \right).$$

Therefore, $t_{V_1}(G_1) = t_{V_2}(G_2)$, $T_{V_1}(G_1) = T_{V_2}(G_2)$, and so $\Delta^t_{V_1}(G_1) = \Delta^t_{V_2}(G_2)$.

□

Theorem 3.20. *If G_1 is a co-weak isomorphic irregular vague graph of G_2, then*
(i) $d_{V_1}(G_1) \leq d_{V_2}(G_2)$ and $D_{V_1}(G_1) \leq D_{V_2}(G_2)$, and so $\Delta_{V_1}(G_1) \leq \Delta_{V_2}(G_2)$.
(ii) $t_{V_1}(G_1) \leq t_{V_2}(G_2)$ and $T_{V_1}(G_1) \leq T_{V_2}(G_2)$, and so $\Delta^t_{V_1}(G_1) \leq \Delta^t_{V_2}(G_2)$.

Proof. (i) Assum that $h : V_1 \to V_2$ is a co-weak isomorphism of G_1 and G_2. If D_1 is a minimal dominating set of G_1, then by Definition 3.14, we get $h(D_1)$ is a minimal dominating set of G_2 such that $O(D_1) \leq O(h(D_1))$. Therefore, $d_{V_1}(G_1) \leq d_{V_2}(G_2)$, $D_{V_1}(G_1) \leq D_{V_2}(G_2)$, and so $\Delta_{V_1}(G_1) \leq \Delta_{V_2}(G_2)$.

(ii) Assum that $h : V_1 \to V_2$ is a co-weak isomorphism of G_1 and G_2. If D_1^t is a minimal total dominating set of G_1, then by Definition 3.14, we get $h(D_1^t)$ is a minimal total dominating set of G_2 such that $O(D_1^t) \leq O(h(D_1^t))$. Therefore, $t_{V_1}(G_1) \leq t_{V_2}(G_2)$, $T_{V_1}(G_1) \leq T_{V_2}(G_2)$, and so $\Delta^t_{V_1}(G_1) \leq \Delta^t_{V_2}(G_2)$.

□

Theorem 3.21. *If G_1 is a co-weak isomorphic irregular vague graph of G_2, then*
(i) $b_{E_1}(G_1) = b_{E_2}(G_2)$ and $B_{E_1}(G_1) = B_{E_2}(G_2)$.

(ii) $b^t_{E_1}(G_1) = b^t_{E_2}(G_2)$ and $B^t_{E_1}(G_1) = B^t_{E_2}(G_2)$.

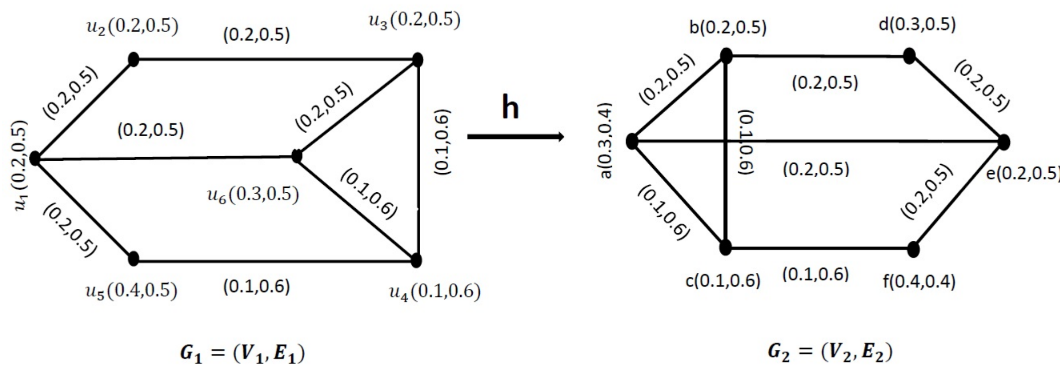

Figure 5: Irregular vague graphs G_1 and G_2.

Proof. (i) Assum that $h : V_1 \to V_2$ is a co-weak isomorphism of G_1 and G_2. If $e = uv$ is an additional strong arc in G_1^*, then by Definition 3.14, we get $e^{'} = h(u)h(v)$, with coordinates $t_{B_1}(e) = t_{B_2}(e')$ and $f_{B_1}(e) = f_{B_2}(e')$, is an additional strong arc in G_2^*. Therefore, $b_{E_1}(G_1) = b_{E_2}(G_2)$ and $B_{E_1}(G_1) = B_{E_2}(G_2)$.

(ii) Assum that $h : V_1 \to V_2$ is a co-weak isomorphism of G_1 and G_2. If $e = uv$ is an additional strong arc in G_1^*, then by Definition 3.14, we get $e^{'} = h(u)h(v)$, with coordinates $t_{B_1}(e) = t_{B_2}(e')$ and $f_{B_1}(e) = f_{B_2}(e')$, is an additional strong arc in G_2^*. Therefore, $b^t_{E_1}(G_1) = b^t_{E_2}(G_2)$ and $B^t_{E_1}(G_1) = B^t_{E_2}(G_2)$.

□

Theorem 3.22. *If $G_1 \cong G_2$, then*
(i) $b_{E_1}(G_1) = b_{E_2}(G_2)$ *and* $B_{E_1}(G_1) = B_{E_2}(G_2)$.

(ii) $b^t_{E_1}(G_1) = b^t_{E_2}(G_2)$ *and* $B^t_{E_1}(G_1) = B^t_{E_2}(G_2)$.

Proof. The proof is straightforward. □

Proposition 3.23. *If $G_1 \cong G_2$, then*
(i) $d^2_{V_1}(G_1) = d^2_{V_2}(G_2)$, $D^2_{V_1}(G_1) = D^2_{V_2}(G_2)$, *and so* $\Delta^2_{V_1}(G_1) = \Delta^2_{V_2}(G_2)$.

(ii) $td^2_{V_1}(G_1) = td^2_{V_2}(G_2)$, $TD^2_{V_1}(G_1) = TD^2_{V_2}(G_2)$, *and so* $T\Delta^2_{V_1}(G_1) = T\Delta^2_{V_2}(G_2)$.

Proof. (i) Assum that $h : V_1 \to V_2$ is an isomorphism of G_1 and G_2. If D_2 is a minimal 2-dominating set of G_1, then by Definition 3.14, we get $h(D_2)$ is a minimal 2-dominating set of G_2 and

$$\sum_{v_i \in D_1} \left(\frac{t_{A_1}(v_i) + (1 - f_{A_1}(v_i))}{2} \right) = \sum_{h(v_i) \in h(D_2)} \left(\frac{t_{A_2}(v_i) + (1 - f_{A_2}(v_i))}{2} \right).$$

Therefore, $d^2_{V_1}(G_1) = d^2_{V_2}(G_2)$, $D^2_{V_1}(G_1) = D^2_{V_2}(G_2)$, and so $\Delta^2_{V_1}(G_1) = \Delta^2_{V_2}(G_2)$.

(ii) Assum that $h : V_1 \to V_2$ is an isomorphism of G_1 and G_2. If D_2^t is a minimal total dominating set of G_1, then by Definition 3.14, we get $h(D_2^t)$ is a minimal total 2-dominating set of G_2 and

$$\sum_{v_i \in D_1} \left(\frac{t_{A_1}(v_i) + (1 - f_{A_1}(v_i))}{2} \right) = \sum_{h(v_i) \in h(D_2^t)} \left(\frac{t_{A_2}(v_i) + (1 - f_{A_2}(v_i))}{2} \right).$$

Therefore, $td^2_{V_1}(G_1) = td^2_{V_2}(G_2)$, $TD^2_{V_1}(G_1) = TD^2_{V_2}(G_2)$, and so $T\Delta^2_{V_1}(G_1) = T\Delta^2_{V_2}(G_2)$.

□

Proposition 3.24. *If G_1 is a co-weak isomorphic irregular vague graph of G_2, then*
(i) $d^2_{V_1}(G_1) \leq d^2_{V_2}(G_2)$, $D^2_{V_1}(G_1) \leq D^2_{V_2}(G_2)$, and so $\Delta^2_{V_1}(G_1) \leq \Delta^2_{V_2}(G_2)$.

(ii) $td^2_{V_1}(G_1) \leq td^2_{V_2}(G_2)$, $TD^2_{V_1}(G_1) \leq TD^2_{V_2}(G_2)$, and so $T\Delta^2_{V_1}(G_1) \leq T\Delta^2_{V_2}(G_2)$.

Proof. (i) Assum that $h : V_1 \to V_2$ is a co-weak isomorphism of G_1 and G_2. If D_2 is a minimal 2-dominating set of G_1, then by Definition 3.14, we get $h(D_2)$ is a minimal 2-dominating set of G_2 such that $O(D_2) \leq O(h(D_2))$. Therefore, $d^2_{V_1}(G_1) \leq d^2_{V_2}(G_2)$, $D^2_{V_1}(G_1) \leq D^2_{V_2}(G_2)$, and so $\Delta^2_{V_1}(G_1) \leq \Delta^2_{V_2}(G_2)$.

(ii) Assum that $h : V_1 \to V_2$ is a co-weak isomorphism of G_1 and G_2. If D_2^t is a minimal total 2-dominating set of G_1, then by Definition 3.14, we get $h(D_2^t)$ is a minimal total 2-dominating set of G_2 such that $O(D_2^t) \leq O(h(D_2^t))$. Therefore, $td^2_{V_1}(G_1) \leq td^2_{V_2}(G_2)$, $TD^2_{V_1}(G_1) \leq TD^2_{V_2}(G_2)$, and so $T\Delta^2_{V_1}(G_1) \leq T\Delta^2_{V_2}(G_2)$.

□

Proposition 3.25. *If $G_1 \cong G_2$, then*
(i) $b^2_{E_1}(G_1) = b^2_{E_2}(G_2)$ and $B^2_{E_1}(G_1) = B^2_{E_2}(G_2)$.

(ii) $tb^2_{E_1}(G_1) = tb^2_{E_2}(G_2)$ and $TB^2_{E_1}(G_1) = TB^2_{E_2}(G_2)$.

Proof. The proof is straightforward. □

Proposition 3.26. *If G_1 is a co-weak isomorphic irregular vague graph of G_2, then*
(i) $b^2_{E_1}(G_1) = b^2_{E_2}(G_2)$ and $B^2_{E_1}(G_1) = B^2_{E_2}(G_2)$.

(ii) $tb^2_{E_1}(G_1) = tb^2_{E_2}(G_2)$ and $TB^2_{E_1}(G_1) = TB^2_{E_2}(G_2)$.

Proof. (i) Assum that $h : V_1 \to V_2$ is a co-weak isomorphism of G_1 and G_2. If $e = uv$ is an additional strong arc in G_1^*, then by Definition 3.14, we get $e' = h(u)h(v)$, with coordinates $t_{B_1}(e) = t_{B_2}(e')$ and $f_{B_1}(e) = f_{B_2}(e')$, is an additional strong arc in G_2^*. Therefore, $b^2_{E_1}(G_1) = b^2_{E_2}(G_2)$ and $B^2_{E_1}(G_1) = B^2_{E_2}(G_2)$.

(ii) Assum that $h : V_1 \to V_2$ is a co-weak isomorphism of G_1 and G_2. If $e = uv$ is an additional strong arc in G_1^*, then by Definition 3.14, we obtain $e' = h(u)h(v)$, with coordinates $t_{B_1}(e) = t_{B_2}(e')$ and $f_{B_1}(e) = f_{B_2}(e')$, is an additional strong arc in G_2^*. Therefore, $tb^2_{E_1}(G_1) = tb^2_{E_2}(G_2)$ and $TB^2_{E_1}(G_1) = TB^2_{E_2}(G_2)$.

□

Theorem 3.27. *If $G_1 \cong G_2$, then $I(G_1) = I(G_2)$.*

Proof. Assum that $h : V_1 \to V_2$ is an isomorphism of G_1 and G_2. If S_1 is a maximal independent set of G_1, then by Definition 3.14, we have $h(S_1)$ is a maximal independent set of G_2 and

$$\sum_{v_i \in S_1} \left(\frac{t_{A_1}(v_i) + (1 - f_{A_1}(v_i))}{2} \right) = \sum_{h(v_i) \in h(S_1)} \left(\frac{t_{A_2}(v_i) + (1 - f_{A_2}(v_i))}{2} \right).$$

Therefore, $i_{V_1}(G_1) = i_{V_2}(G_2)$ and $I_{V_1}(G_1) = I_{V_2}(G_2)$, and so $I(G_1) = I(G_2)$. □

Theorem 3.28. *If G_1 is a co-weak isomorphic irregular vague graph of G_2, then $I(G_1) \leq I(G_2)$.*

Proof. Assum that $h : V_1 \to V_2$ is a co-weak isomorphism of G_1 and G_2. If S_1 is a maximal independent set of G_1, then by Definition 3.14, we get $h(S_1)$ is a maximal independent set of G_2 such that $O(S_1) \leq O(h(S_1))$. Therefore, $i_{V_1}(G_1) \leq i_{V_2}(G_2)$ and $I_{V_1}(G_1) \leq I_{V_2}(G_2)$, and so $I(G_1) \leq I(G_2)$. □

4 Application

The theory of vague graphs has many applications in new sciences and technologies. Now days, the concepts of dominating sets, cobondage sets, and numbers are considered as the fundamental concepts in the theory of vague graphs and have applications in several fields, especially in the fields of operations research, neural networks, electrical networks and monitoring communications. By comparing the definition of the concept of irregular vague graphs in this paper and the definition of the concept of regular vague graphs in [8], we find that the class of irregular vague graphs is much wider than the class of regular vague graphs, so studing and investigating of the concepts of dominating sets, cobondage sets, dominating numbers and cobondage numbers is particularly important in the class of irregular vague graphs. In [3, 4], using the concept of additional strong arc, a model was proposed to reduce vertex cardinality of dominating set and 2-dominating set, while increasing optimal effective weight of vague graph. What is now considered is the enhancement and increasement of domination number and 2-domination number parameters of an irregular vague graph and its apparent deformation, while remaining stability of the cobondage number and 2-cobondage number parameters and its inherent pattern by meaning of the concepts presented in the isomorphic images of an irregular vague graph.

4.1 Locating fire stations and emergency medical centers in urban regions of the metropolis (based on a fixed inherent model and optimal assurance of non-accidental variability).

The location map of fire stations and emergency medical centers in each urban regions of a metropolis can be considered as an irregularly vague graph. In these graphs, the vertices indicate that the Fire Stations (F.S) and the emergency medical center (E.M.C) and edge indicate their communication paths in these regions. We define the values of f-strength and t-strength for any $v \in V$ and $e \in E$, as follows:
$t_A(v)$: The minimum assurance of non-incidentalism in mission scope of v.
$f_A(v)$: The minimum assurance of incidentalism in mission scope of v.
$t_B(e)$: The minimum assurance of the timely presence at the incident scene through the e path.
$f_B(e)$: The minimum assurance of the timely absence at the incident scene through the e path.
Thus, the size of each vertex stands for $|v| = \dfrac{1 + t_A(v) - f_A(v)}{2}$, for any $v \in V$, represents the optimal level of assurance of the non-incidentalism of that region. Also, the size of each edge

stands for $|e| = \dfrac{1 + t_B(e) - f_B(e)}{2}$, represents the optimal level of assurance of timely presence in the incident scene through that edge (path). It should be noted that such things as urban texture, the driving culture and the extent to which drivers and pedestrians follow driving rules, the type of industry and the presence of high-risk industries in one scope, etc., are factors that contribute to the estimation of the incidentalism or non-incidentalism in that scope. Also, the factors such as the volume of traffic, the number of traffic lights, the squares, the overpasses and pedestrian underpasses and the maximum and minimum speed of vehicles per path, etc., are affected by the route in the estimation of the assurance of timely presence or absence in that path. According to the above comparison, the dominating sets could be considered as the location of fire stations and medical emergency centers in the city.

Therefore, locating fire stations and emergency medical centers as dominating set or 2-dominating set in each urban regions of a metropolis based on a fixed inherent model and pattern, as well as a map of constant optimization when necessary (cobondage set or 2-cobondage set) in decision-making in various urban development issues, passive defense issues in a metropolis, and so on are of great importance, and the application of concepts and findings related to isomorphic images of an irregularly vague graph plays an important role in achieving this goal. For example, in the Figure 6, the G vague graph and co-weak isomorphic images are examined as locating maps of fire stations and medical emergency centers in urban regions of a metropolis. It is noteworthy that the number of strong arcs as well as the number of vertices of dominating sets are all constant and only the standard size of each vertex has changed according to cultural, social, urban, industrial, etc, requirements.

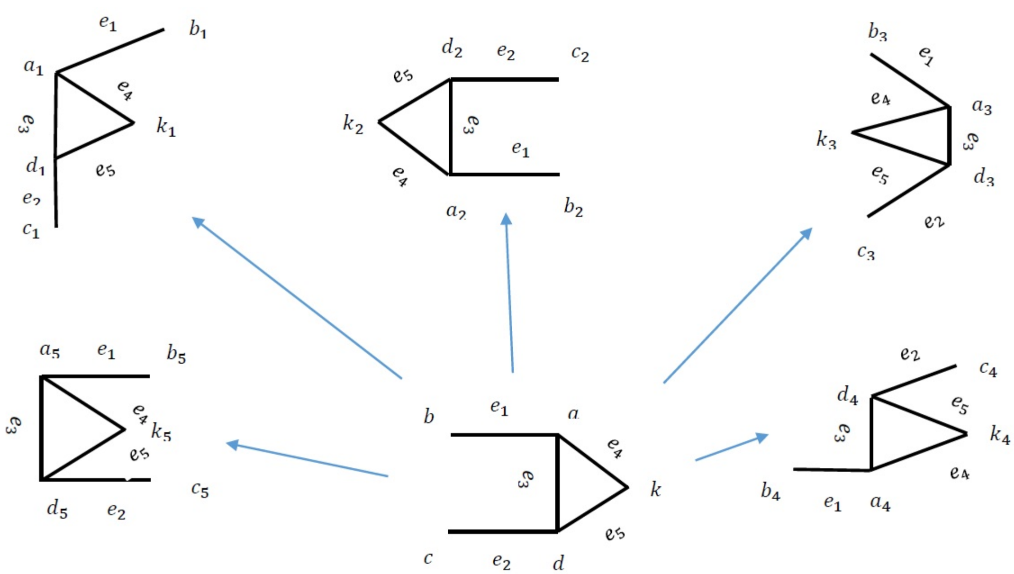

Figure 6: Vague graph G and co-weak isomorphic images of G.

	a	b	c	d	k	Δ_V	b_E
C	(0.2,0.5)	(0.2,0.5)	(0.2,0.5)	(0.2,0.5)	(0.2,0.5)	0.875	0.35
C_1	(0.2,0.4)	(0.2,0.5)	(0.3,0.5)	(0.2,0.5)	(0.2,0.5)	0.9	0.35
C_2	(0.2,0.5)	(0.2,0.5)	(0.2,0.4)	(0.3,0.5)	(0.2,0.5)	0.925	0.35
C_3	(0.3,0.4)	(0.2,0.5)	(0.2,0.5)	(0.2,0.5)	(0.2,0.5)	0.875	0.35
C_4	(0.2,0.5)	(0.2,0.5)	(0.3,0.4)	(0.2,0.5)	(0.2,0.5)	0.925	0.35
C_5	(0.2,0.5)	(0.2,0.5)	(0.2,0.5)	(0.2,0.4)	(0.3,0.4)	0.925	0.35

5 Conclusion

The theory of vague graphs has many applications in new science and technology. Since the vague models compare the classical and fuzzy models to the system, they give more accuracy, flexibility and compatibility. In this paper, irregular and irregular edges and some of its variants are presented and examined. Also discussed some special conditions in which irregularities are matched together are discussed. Finally, by using isomorphic images of an irregularly vague graph, a model for optimizing the domination number and 2-domination number parameters was presented, while unlike the model presented in [3], the number of vertices of dominating set as well as the cobondage number and 2-cobondage number parameters remain constant.

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